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Iapetus Hypothetical Sub-Satellite Re-Visited and it Reveals Celestial Body Formation Process in the Primary-Centric Framework

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Supplementary Material

Appendix A

Kinematic Model Analysis of the evolutionary history of Saturn-Iapetus System.

A.1. Derivation Of Ω/Ω=Lom/Lod Equation Using Keplerian Approximation.

LOM=length of month $= P_1$ =sidereal orbital period of the Satellite in question (Iapetus in the present case) $= 2\pi/\Omega$;

LOD=length of day= P_2 =sidereal spin period of the Planet in question (Saturn in the present case) $= 2\pi/\omega$.

Kepler's Third Law:

$$a^3\Omega^2 = G\left(M_{Sat} + M_{Iap}\right) \qquad A.1$$

From Eq. (A.1) we obtain:

$$\Omega \times a^2 = \sqrt{\left(G\left(M_{Sat} + M_{Iap}\right)a\right)} \qquad A.2$$

Let $B = \sqrt{G(M_{Sat} + M_{Iap})} \quad m^{3/2}/sec$

Then $\Omega \times a^2 = B\sqrt{a}$ $\quad A.3$

$$\Omega = \frac{B}{a^{3/2}} \quad A.4$$

Or

Let total angular momentum of Saturn-Iapetus be J_T defined as follows:

From the law of conservation of angular momentum in absence of any external torque,

$$J_T = J_{spinSat} + J_{spinIap} + J_{orb} = constant \quad A.5$$

$$J_T = 0.4M_{Sat}R_{Sat}^2 \times \left(\frac{2\pi}{P_{spinSat}}\right) + 0.4M_{Iap}R_{Iap}^2 \times \left(\frac{2\pi}{P_{spinIap}}\right) + \frac{M_{Iap}}{1+\frac{M_{Iap}}{M_{Sat}}} a_{Iap}^2 \times \left(\frac{2\pi}{P_{orbit}}\right) \quad A.6$$

$$Orbital\ Angular\ Momentum = J_{orb} = \left[\frac{M_{Iap}}{1+\frac{M_{Iap}}{M_{Sat}}} \times a_{Iap}^2 \times \Omega\right] \frac{Kg - m^2}{s} \quad A.7$$

$$Spin\ Angular\ Momentum = J_{spinSat} = 0.4M_{Sat}R_{Sat}^2 \times \left(\frac{2\pi}{P_{spinSat}}\right) = C\omega\ \frac{Kg - m^2}{s} \quad A.8$$

Where,

C=moment of Inertia of Saturn around its spin axis=0.4M_Sat R_Sat^2;

RSat=mean radius of Saturn, ω=spin angular velocity of Saturn = $\frac{2\pi}{P_{spin\_Sat}}$;

Spin Angular Momentum of Iapetus:

$$J_{spinIap} = 0.4M_{Iap}R_{Iap}^2 \times \left(\frac{2\pi}{P_{spinIap}}\right) = I_{Iap} \times \left(\frac{2\pi}{P_{spinIap}}\right) \frac{Kg - m^2}{s} \quad A.9$$

Where R_{Iap}=mean radius of Iapetus and $P_{SpinIap}$=spin period of Iapetus around its spin axis and I_{Iap}=moment of inertia of Iapetus around its spin axis.

In most close binaries, the satellite is in captured rotation or synchronous orbit which implies that Satellite is tidally locked with the central planet. So Iapetus is also tidally locked with_Saturn_and

$\omega_{spinIap} = \Omega = orbital\ angular\ velocity\ all\ the\ time\ therefore\ Eq. A.6\ reduces\ to$:

$$J_T = C\omega_{spinSat} + \left(I_{Iap} + \frac{M_{Iap}}{1+\frac{M_{Iap}}{M_{Sat}}} \times a_{Iap}^2\right)\Omega \qquad A.10$$

Equation A.10 is further simplified:

$$J_T = C\omega_{spinSat} + \left(\frac{I_{Iap}}{a_{Iap}^2} + \frac{M_{Iap}}{1+\frac{M_{Iap}}{M_{Sat}}}\right)\Omega \times a_{Iap}^2 \qquad A.11$$

In most cases $a_{Iap} >> R_{Iap}$ hence equation A.11 is simplified to:

$$J_T = C\omega_{spinSat} + \left(\frac{M_{Iap}}{1+\frac{M_{Iap}}{M_{Sat}}}\right)\Omega \times a_{Iap}^2 \qquad A.12$$

But this simplification will not be valid when the secondary is in sub-synchronous orbit and it is spiraling-in as it is in hot-Jupiter case.

Substituting Eq. (A.3) in Eq. (A.12), we get:

$$J_T = C\omega_{spinSat} + \left(\frac{M_{Iap}}{1+\frac{M_{Iap}}{M_{Sat}}}\right)B\sqrt{a_{Iap}} \qquad A.13$$

Rearranging the terms, we get:

$$\frac{2\pi}{LOD} = \frac{2\pi}{P_{spinSat}} = \omega_{spinSat} = \frac{J_T}{C} - \left(\frac{M_{Iap}}{1+\frac{M_{Iap}}{M_{Sat}}}\right)\frac{B}{C}\sqrt{a_{Iap}} \qquad A.14$$

LOD is sidereal length of day and LOM is sidereal length of month.

Dividing equation A.14 by equation A.3 we get:

$$\frac{LOM}{LOD} = \frac{\omega_{spinSat}}{\Omega} = E \times a_{Iap}^{3/2} - F \times a_{Iap}^2 \qquad A.15$$

Where $E = \frac{J_T}{BC}$ and $F = \left(\frac{M_{Iap}}{1+\frac{M_{Iap}}{M_{Sat}}}\right)\frac{1}{C}$;

LOM/LOD is the nomenclature adopted in Earth-Moon System and today its value is 27.3 in case of Earth-Moon system.

The key equation in Planetary Satellite Dynamics is ω/Ω or LOM/LOD equation which will be used to arrive at the correct form of radial velocity of recession/approach. The time integral of the reciprocal of radial velocity will give us the evolution of satellite's semi-major axis with time.

Equation (A.15) equated to Unity:

$$\frac{LOM}{LOD} = \frac{\omega_{spinSat}}{\Omega} = E \times a_{Iap}^{3/2} - F \times a_{Iap}^{2} = 1 \qquad A.16$$

Eq.(A.16) has two roots: a_{G1} and a_{G2} which for Earth-Moon System is known as inner and outer geo-synchronous orbits and in general binary case these are known as inner and outer Clarke's orbits.

In the region between aG1 and a_{G2}, lom/lod is greater than Unity and within a_{G1}, lom/lod is less than Unity. Beyond a_{G2} lom/lod is negative which makes it physically untenable and hence the region beyond a_{G2} is a forbidden zone. That is if satellite is gravitationally bounded to central body it can never spiral beyond a_{G2}.

Inner Clarke's Orbit is an energy maxima (Appendix B) hence it is an unstable equilibrium orbit whereas outer Clarke's Orbit is an energy minima (Appendix B) of the binary system. Hence a secondary at a_{G1} tumbles out of the orbit at the slightest perturbation due to solar wind, cosmic particles or radiation pressure.

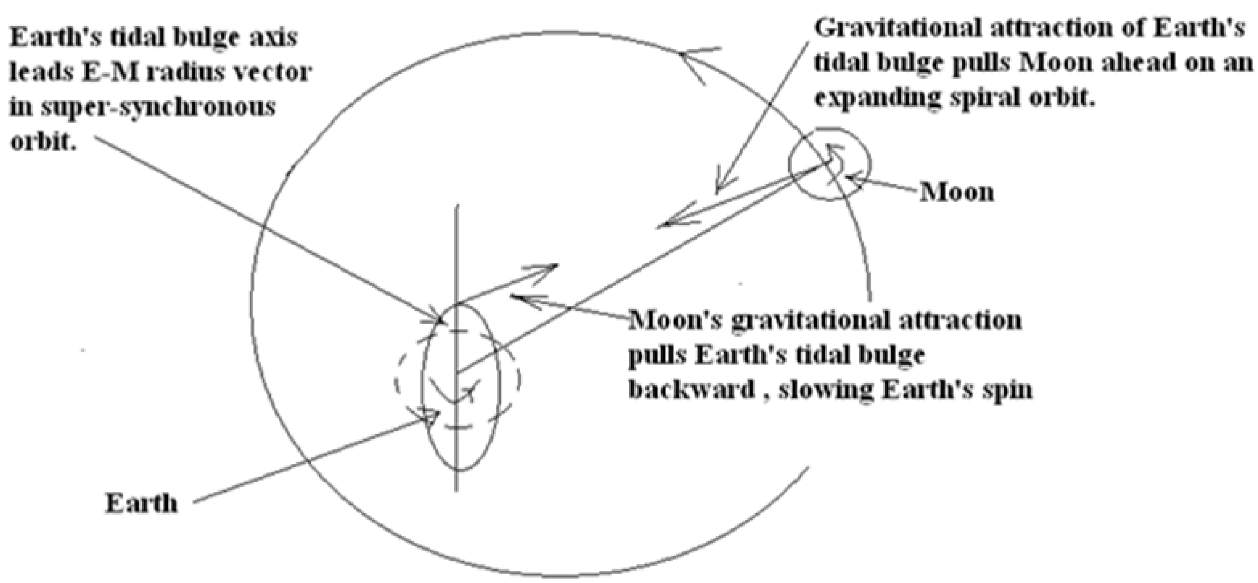

Figure A1: In Earth-Moon System, Moon is in super-synchronous orbit. The of-setting of the line of bulge in Earth with respect to E-M radius vector creates a tidal drag and de-spinning of Earth leading to secular lengthening of day. The de-spinning of Earth leads to increased angular momentum of Moon. During the conservative phase of the evolution of E-M sysem by gravitational sling shot impulsive torque Moon is launched on an expanding spiral path around Earth. After the conservative phase, Earth coasts on its own towards the outer Geosynchronous orbit where it terminates its non-keplerian journey.

The secondary which falls long of a_{G1} is in super-synchronous orbit. This is the case with Moon. Moon orbits in 27.3 days and Earth spins in 1 day. Hence Earth's tidal bulge leads the Earth-Moon radius vector as shown in Figure A.1. This results in a tidal drag on Earth which leads to secular lengthening of our diurnal day. Length of Day has increased from 5 hours to 24 hours over a period of 4.467Gy, the age of Earth and Moon. This simultaneously leads to the spiraling out of Moon. Moon was formed at 18,000Km just beyond Roches' limit. Today it is at 384,400Km from Earth.

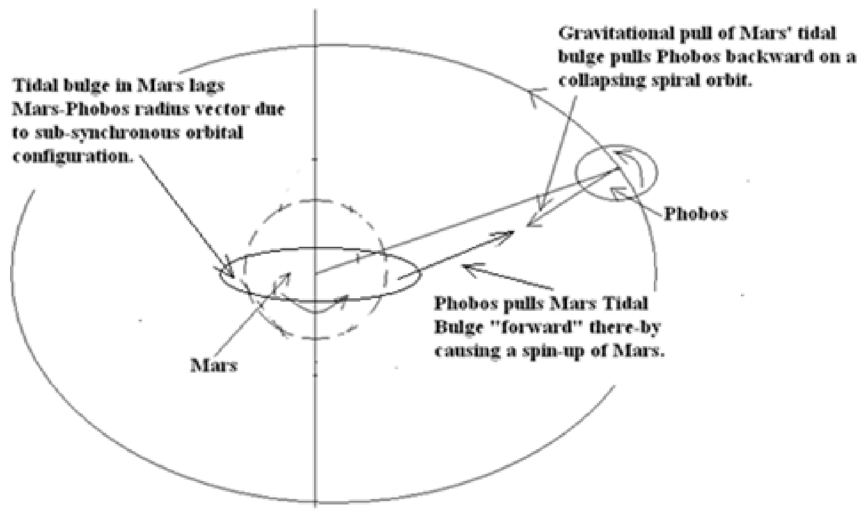

Figure A2: In Mars-Phobos-System, Phobos is in sub-synchronous orbit and is speeding up Mars spin and Phobos is losing its angular momentum and its rotational kinetic Energy and angular momentum and rotational kinetic energy is correspondingly being increased in Mars so as to conserve the total angular momentum and total rotational energy of the system. In the process Phobos is launched on a gravitationally runaway collapsing spiral orbit. Here the tidal bulge in Mars is lagging the radius vector joining Mars and Phobos hence sub-synchronous Phobos is spinning up Mars.

The secondary which falls short of a_{G1} is in sub-synchronous orbit. This is the case with Phobos, a moon of Mars. Phobos orbits in 0.319 day and Mars spins in 1.02596 day. Hence Mars' tidal bulge axis lags the Mars-Phobos radius vector as shown in Figure A.2. This results in a tidal acceleration of Mars which leads to secular shortening or spin-up of Mars diurnal day. Length of Day *i.e.* spin-period of Mars has reduced from 1.0263d to 1.02596d. This simultaneously leads to the spiral-in of Phobos from 20,432Km to the present orbit of 9,377.2 Km. According to my calculations ASCOM-D-20-00010 Phobos is losing altitude at the rate of 18.29 cm per year and in next 10My it will merge with Mars.

When the secondary tumbles into a super-synchronous orbit, it experiences a powerful impulsive torque due to gravitational sling shot effect. This is the case with Earth-Moon.

A.2. The phenomena of gravitational slingshot.

Planet fly-by, gravity assist is routinely used to boost the mission spacecrafts to explore the far reaches of our solar system [Dukla, Cacioppo & Gangopadhyaya 2004, Jones 2005, Epstein 2005, Cook 2005]. Voyager I and II used the boost provided by Jupiter to reach Uranus and Neptune. Cassini has utilized 4 such assists to reach Saturn.

A space-craft which passes "behind" the moon gets an increase in its velocity (and orbital energy) relative to the primary body. In effect the primary body launches the space craft on an outward spiral path. If the spacecraft flies "infront" of a moon, the speed and the orbital energy decreases. Traveling "above" and "below" a moon alters the direction modifying only the orientation (and angular momentum magnitude). Intermediate flyby orientation change both energy and angular momentum. Accompanying these actions there are reciprocal reactions in the corresponding moon.

The above slingshot effect is in a three body problem. In a three body problem, the heaviest body is the primary body. With respect to the primary body the secondary system of two bodies are analyzed.

In case of planet flyby, planet is the primary body and the moon- spacecraft constitute the secondary system.

While analyzing the planetary satellites, Sun is the primary body and planet-satellite is the secondary system. But in our Keplerian approximate analysis, Sun has been neglected without any loss of generality and without any loss of accuracy. In fact the general trend of evolution of our Moon has been correctly analyzed using this approach.

While analyzing the Sun-Planet system, galactic center is the primary body and Sun-Planet is the secondary system. While analyzing our solar system or exo-solar system, the galactic center has been neglected and we have essentially analyzed Sun-Planet as a two body problem.

In a similar fashion in the analysis of Planet Flyby-Gravity Assist Maneuvers, Planet is the primary body. The planet can be neglected and moon-spacecraft can be treated as a two body problem and the same results can be obtained without any loss of accuracy or generality.

The gravitational sling shot becomes clearer if we look at the radial acceleration and radial velocity profile.

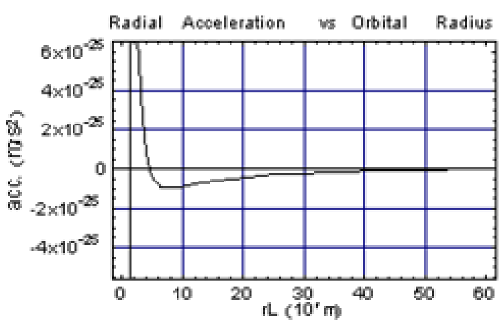

Figure A3: Radial Acceleration Profile of Moon (Within aG1 the Moon is accelerated inward. Beyond aG1 the Moon is rapidly accelerated outward under the influence of an impulsive gravitational torque due to rapid transfer of spin rotational energy. The maxima of the outward radial acceleration occurs at a1. (This is the peak of the impulsive sling shot torque.)

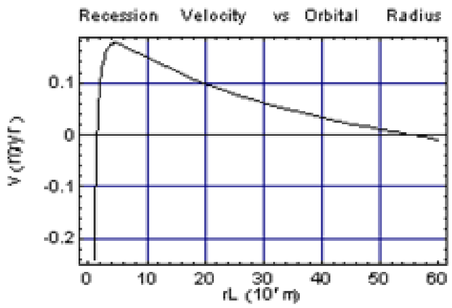

Figure A4: Radial Velocity Profile of Moon. (Beyond aG1, Moon is rapidly accelerated to a maximum radial velocity,Vmax, at a2 where Sling-Shot Effect terminates and radial acceleration is zero. Then onward Moon coasts on it own towards the outer Geo-Synchronous Orbit aG2).

A.3: Setting up of the time integral equation.

From equation A.7, the angular momentum of Iapetus-Saturn system is:

$$J_{orb} = \frac{M_{Iap}}{1+\frac{M_{Iap}}{M_{Sat}}} \times a_{Iap}^2 \times \Omega_{Iap} \qquad A.7$$

Substituting equation A.3 in equation A.7 we get:

$$J_{orb} = \frac{M_{Iap}}{1+\frac{M_{Iap}}{M_{Sat}}} \times B a_{Iap}^{1/2} \qquad A.17$$

Tidal Torque exerted by Iapetus on Saturn is the rate of change of spin angular momentum of Saturn=Tidal Torque exerted by Saturn on Iapetus which is the rate of change of orbital angular momentum. This comes directly from conservation of the total angular momentum of Saturn-Iapetus System in absence of any external torque.

Hence differentiating equation A.17 we get:

$$Torque = \frac{dJ_{orb}}{dt} = m^* B \frac{1}{2a_{Iap}^{1/2}} \times \frac{da_{Iap}}{dt} \qquad A.18$$

Where $m^* = reduced\ mass\ of\ Iapetus = \frac{M_{Iap}}{1+\frac{M_{Iap}}{M_{Sat}}}$

Since this tidal torque is a product of (ω/Ω-1) and structure function (K/a_{Iap}^{M}) where K=structure constant and M=structure exponent.

So the empirical form of this tidal Torque is taken as:

$$Torque = \frac{K}{a_{Iap}^{M}}\left(\frac{\omega}{\Omega} - 1\right) \qquad A.19$$

Where $\frac{\omega}{\Omega} = E \times a_{Iap}^{3/2} - F \times a_{Ia}^{2}$ $\qquad$ *from Eq. A.15*

Where

$E = \frac{J_T}{BC_O}$ and $F = \frac{M_{Ia}}{1+\frac{M_{Ia}}{M_{Sat}}} \times \frac{1}{C_O}$ and C_O = *Moment of Inertia of Saturn around spin axis.*

We have already seen that Eq. A.16 has two roots: a_{G1} and a_{G2} known as inner and outer Clarke's orbit at triple synchrony point where ω/Ω=1. These are triple synchrony orbits where the two bodies are tidally interlocked and Spin Period of Saturn=Spin Period of Iapetus=Orbital Period of Iapetus. In all binaries systems a_{G1} is unstable equilibrium orbit and a_{G2} is stable equilibrium orbit. All binaries start with aG1 seperation. The secondary always tumbles out of a_{G1}. If it tumbles long of a_{G1} the separation will increase to a_{G2} and no more. Secondary can never go beyond a_{G2}. Either the secondary remains stay put in outer Clarke's Orbit or it is deflected back on a collapsing spiral orbit. If the secondary falls short of a_{G1} the binary is bound to collapse to merger of the two components of the binary.

Equating A.18 and A.19 we get an expression for radial velocity of Iapetus which can be receding or approaching depending on the orbital configuration of Iapetus.

$$m^{*}B\frac{1}{2a_{Iap}^{1/2}} \times \frac{da_{Iap}}{dt} = \frac{K}{a_{Iap}^{M}}\left(\frac{\omega}{\Omega} - 1\right) \qquad A.20$$

Rearranging the terms we get the expression of the radial velocity:

$$\frac{da_{Iap}}{dt} = \frac{K}{a_{Iap}^{M}}\left(\frac{\omega}{\Omega} - 1\right) \times \frac{2a^{1/2}}{m^{*}B} = \frac{K}{a_{Iap}^{M}}\left(E \times a_{Iap}^{3/2} - F \times a_{Iap}^{2} - 1\right) \times \frac{2a^{1/2}}{m^{*}B}$$

Or

$$v(a_{Iap}) = \frac{da_{Iap}}{dt} = \frac{2K}{a_{Iap}^{M}} \times \frac{1}{m^{*}B} \times \left(E \times a_{Iap}^{2} - F \times a_{Iap}^{2.5} - \sqrt{a_{Iap}}\right) \qquad A.21$$

Between a_{G1} and a_{G2}, ω/Ω is greater than Unity hence radial velocity is positive and recessive.

At less than a_{G1}, ω/Ω is less than Unity hence radial velocity is negative and secondary approaches primary.

At greater than a_{G2}, ω/Ω is negative which is physically not possible in a prograde system hence system is untenable and it is a forbidden state.

Spin to Orbital velocity equation yields a root when it is in second Mean Motion Resonance (MMR) position. That is:

$$\frac{\omega}{\Omega} = E \times a_{Iap}^{3/2} - F \times a_{Iap}^{2} = 2 \qquad A.22$$

This gives a root at a2 which is gravitation resonance point and I assume that after the secondary undergoes gravitational sling shot impulsive torque, it attains maximum recession velocity at this point. After this maxima, recession velocity continuously decreases until it reaches zero magnitude at outer Clarke's Orbit.

Thus as is evident from equation A.21, recession velocity is zero at a_{G1} and a_{G2}. From a_{G1} to a_2, the system is in conservative phase and secondary experiences a powerful sling-shot impulsive torque which imparts sufficient rotational energy to the secondary by virtue of which the secondary coasts on its own from a_2 to a_{G2} during which time the system is in dissipative phase, Secondary is exerting a tidal drag on the central body and all the rotational energy released by the central body as a result of de-spinning is lost as tidal heat, but not completely. This tidal heat is produced during tidal stretching and squeezing of the primary and it may be produced in secondary if secondary is not in synchronous orbit or if it is in synchronous orbit but it is in eccentric orbit. A dissipative orbit is never stable and the secondary spirals out from inner Clarke's Orbit to the outer Clarke's Orbit.

When the secondary tumbles into sub-synchronous orbit it experiences a negative radial velocity which launches it on a collapsing spiral and the system is spun-up. In this collapsing phase, secondary exerts an accelerating torque on the central body and rotational energy is transferred to the primary. This rotational energy causes spin-up of the central body as well as it tidally heats up the central body by tidal deformations. This spiral-in is also dissipative hence unstable leading to ultimate merger of the two bodies.

Since equation A.21 has a radial velocity maxima at a2 therefore the first derivative of equation A.21 has a zero at a2. Equating the first derivative of equation A.21 to zero we get:

$$E(2-M)a_{Iap}^{1.5} - F(2.5-M)a_{Iap}^{2} - (0.5-M) = 0 \; at \; a_2 \qquad A.23$$

From equation A.23, structure exponent 'M' is calculated.

We donot yet know the structure constant K. We make an intelligent guess of V_{max} and calculate the value of 'K' from equation A.21equated to Vmax at semi-major axis 'a_2'.

Using these values of 'K' and 'M' the time integral equation is set up and tested for the age of the system.

$$\int [\frac{1}{v(a_{Iap})} da, a_{G1}, a_{Iapresent}] = transit\ time\ from\ a_{G1}\ to\ the\ present\ value\ of\ a_{Iap} \quad A.24$$

This transit time should be of the order of 4.5Gy in the case of Iapetus because that is the age of Iapetus. [Castillo- Rogez et al (2007)]. Through several iterations we arrive at the correct value of K.

Roche's Limit is given by the formula:

$$a_{Roche} = 2.43\left(\frac{\rho_{Sat}}{\rho_{Ia}}\right)^{1/3} R_{Sat} \qquad A.25$$

Substituting the numerical values of the parameters (ρ_{Sat} = 710Kg/m^3 and ρ_{Iap} = 1083Kg/m^3) in Eq.A.25 we get:

$$a_{Roche} = 1.27 \times 10^8 m$$

Substituting the numerical values of the parameters (ρ_{Sat}=710Kg/m^3 and ρ_{Iap}=1083Kg/m^3) in equation A.25 we get:

$$a_{Roche} = 1.27 \times 10^8 m$$

Therefore we take the initial semi-major axis as a_{ini}=1.28×10^{8}m. This initial semi-major axis corresponds to 12.971 hours orbital period which is the spin period of Iapetus since it is in synchronous orbit due to tidally locked condition. At the time of impact Iapetus was in molten stage and hence it assumed hydrostatic equilibrium ellipsoidal shape corresponding to 12.971 hours spin of Iapetus. This initial semi-major axis ensures that it is beyond Roche's limit hence its status as an accreted body is ensured and it is in super-synchronous orbit (because a_{G1}=1.1×10^{8}m) which ensures that it is de-spinning the whole system in the process Iapetus getting de-spun itself.

A.4. Kinematic Model based analysis of Saturn-Iapetus System.

Table A.1 gives the globe-orbit- spin parameters of Saturn-Iapetus system.

parameters	**magnitude**	**comments**	**Ref**
M_{Sat} (Kg)	5.69×10^{26}		Chaisson, et al.
R_{Sat} (m)	60.268×10^6	Equatorial radius	Chaisson, et al.
Densityρ_{Sat} (Kg/m^3)	710		
P_{Spin_Sat} (d)	0.43(=10.32hours)		Chaisson,et al.
M_{Iap} (Kg)	1.8×10^{21}		Roatsch T, et al.
R_{Iap} (m)	(735.6±3)×10^3		Thomes PC, et al.
Iapetus Dynamical Properties.			
Semi-axis a_{Iap} (m)	3.5613×10^9		Yoder CF, et al.
Semi-axis a_{Iap}(R_{sat})	59.09		
Biaxial ellipsoid Radii(m)	[(747.4±3.1)×10^3]× [(712.4±2)×10^3]		Yoder CF, et al.
(Max-Min)radii(m)	(35±3.7)×10^3	(a-c)/(b-c)=0.95	Thomes PC, et al.

Density, ρ_{Iap} (Kg/m^3)	1083±13		Thomes PC, et al.
P_{orbit} (d)	79.330183	Iapetus orbiting around Saturn	Yoder CF, et al.
Orbital Rate(rad/sec)	9.1670093×10^{-7}		Yoder CF, et al.
P_{spin_Iap} (d)	79.330183	Captured rotation	Yoder CF, et al.
Spin rate(rad/sec)	9.1670093×10^{-7}		Yoder CF, et al.
e (eccentricity)	0.0282		Yoder CF, et al.
α(Inclination)degrees	7.52		Yoder CF, et al.

Using equation A.15 and the parameters given in Table A.1 we determine the kinematic parameters of the Saturn-Iapetus binary system and set up spin to orbital velocity equation.

Substituting the parameters from the Table A.1 in the relevant equations we get:

$$J_T = 1.3983260521 \times 10^{38} \text{ Kg.m}^2.\text{sec}^{-1} \quad \text{A.26}$$

$$C_{Sat} = 0.4 M_{Sat} R_{Sat}^2 = 8.266959631 \times 10^{41} \text{Kg.m}^2 \quad \text{A.27}$$

$$B = \sqrt{[G(M_{Sat} + M_{Iap})]} = 194.857615 \times 10^6 \ (\text{m}^{3/2}/\text{sec}) \quad \text{A.28}$$

$$E = J_T/(BC_{Sat}) = 8.680510156 \times 10^{-13} \text{ m}^{-3/2}; \quad \text{A.29}$$

$$F = (M_{Iap}/(1+M_{Iap}/M_{Sat}))(1/C_{Sat}) = 2.177335304 \times 10^{-21} \text{ m}^{-2}; \quad \text{A.30}$$

$$(\omega/\Omega) = E \times a^{3/2} - F \times a^2 = 184.355 \ (\text{calculated value at } a = 3.56 \times 10^9 \text{ m}); \quad \text{A.31}$$

$$\text{Observed value } (\omega/\Omega) = P_{orbit}/P_{spin} = 184.467;$$

Generally in close binaries the secondary will be tidally locked with the primary. This is also known as captured rotation and the orbit is known as synchronous orbit. This means that secondary's spin angular velocity is kept equal to secondary's orbital angular velocity. This may not be true in wide-orbits.

The roots of the spin to orbital equation:

$$\omega/\Omega = E \times a^{3/2} - F \times a^2 = 1 \quad \text{are:}$$

$$a_{G1}(\text{first Clarke's Orbit}) = 1.1 \times 10^8 \text{ m}; \quad \text{A.32}$$

$$a_{G2}(\text{second Clarke's Orbit}) = 1.58942 \times 10^{17} \text{ m}; \quad \text{A.33}$$

$$€ \ (\text{evolution factor of the secondary}) = (a_{Iap} - a_{G1})/(a_{G2} - a_{G1}) = 2.17 \times 10^{-8} \quad \text{A.34}$$

$$(\text{Time-constant of evolution}) \ \tau = (a_{G2} - a_{G1})/v_{max} \ (\text{m/yr}) = 1.074 \times 10^{16} \text{ yr} \quad \text{A.35}$$

Table A.2: The Kinematic Parameters of Saturn-Iapetus system for studying the evolutionary history of Iapetus.(Calculations have been done using Mathematica).

parameter	magnitude	ref
B(m$^{3/2}$/s)	1.94857615×10^8	App.A
$C_{Sat}=C_O$(kg-m^2)	8.266959631×10^{41}	App.A

a_{ini}(m)	1.28×10^{8}	App.A
a_2(m)	1.74448×10^{8}	App.A
E ($m^{-3/2}$)	$8.680510156\times10^{-13}$	App.A
F (m^{-2})	$2.177335304\times10^{-21}$ m^{-2}	App.A
a_{G1} (m)	1.1×10^{8}	App.A
a_{G2} (m)	1.58942×10^{17}	App.A
M(structure exponent) Dimensionless	3.49997	Eq.A10
K(structure constant) (N-m^{M+1})	4.36391×10^{47}	By iteration
Age=Transit time(Gy)	4.5	Castillo
v_{max} (m/yr)	14.8	By iteration
$v_{present}$(m/yr)	0.32	App.C
Time –constant (yr)	1.074×10^{16}	App.C.

Setting up equation A.24 (the time integral equation), the transit time of Iapetus required for spiraling out from its initial orbit of $a_{ini}=1.28 \times 10^8$ m to a multiple of 1.28×10^8 m is determined and tabulated in Table A.3. The multiple is from 1 to 28. The analysis from multiple 0 to 27.82 covers the entire evolutionary history from 0 to 4.5Gy.

Using Kepler's Third Law equation A.1, the spin of Iapetus in different evolutionary epoch is determined:

Using equation A.36 the de-spun period of Iapetus is determined at multiples of 1.28×10^8 m from 1 to 28.

Using equation 8 of the main text and equation A.36, the dilated values of semi-major axis of the synchronous orbit or triple synchrony orbit of SS are determined in the subsequent ages till the modern times and plotted in Figures A.5 to A.12. The orbit around Iapetus in which SS orbits in 12.971 hours is the triple synchrony orbit at the time of inception of SS as accreted body. These triple synchrony orbits are being referred to as synchronous orbits, a_{syn}SS, by Levison et al. Infact these synchronous orbits converge to Inner Clarke's Orbit as q approaches '0' and they approach Outer Clarke's Orbit as q approaches UNITY. At q=1, Outer Clarke's Orbit falls short of convergence. Synchronous orbit at q=1 is $3.22R_{Iap}$ whereas $a_{G2}=3.92\ R_{Iap}$. Hence KM analysis reduces to Keplerian Analysis at vanishingly small 'q' and convergence just fall short of at q=1. So KM satisfies 'Correspondence Principle' and corresponds to Keplerian Results in Correspondence Limits of 'q' approaching '0'. This will be shown in Appendix C.

All the three values (the evolutionary history of a_{Iap}(×1.28 × 10^8 m), P_{spin_Iap}(d) and asyn(×R_{Iap}) of SS) are tabulated in Table A.3.

Col.1- Age of Iapetus(y), Col.2- a_{Iap}(×1.28×10^8m),Col.3- P_{spin_Iap}(d),Col.4- a_{syn}(×R_{Iap})_q=0,Col.5- a_{syn}(×R_{Iap})_q=0.001,Col.6- a_{syn}(×R_{Iap})_q=0.021,Col.7- a_{syn}(×R_{Iap})_q=0.04,Col.8- a_{syn}(×R_{Iap})_q=0.2,Col.9- a_{syn}(×R_{Iap})_q=0.4,Col.10- a_{syn}(×R_{Iap})_q=0.8;

Table A.3: The expansion of Orbital Radius of Iapetus (a_{Iap})in meter, de-spinning of Iapetus in days and the dilation of sync Orbit of SS as multiples of Iapetus mean radius with time over its evolutionary history of 4.5Gy for different mass ratios of q=SS/Iapetus.

Col.1	**Col.2**	**Col.3**	**Col.4**	**Col.5**	**Col.6**	**Col.7**	**Col.8**	**Col.9**	**Col.10**
0	1	0.54	2.55	2.56	2.57	2.59	2.71	2.86	3.11
1.68M	1.156	0.67	2.95	2.954	2.97	2.99	3.14	3.30	3.59
9.53M	2	1.53	5.11	5.1105	5.14	5.18	5.43	5.72	6.21
23.06M	3	2.81	7.66	7.666	7.72	7.76	8.14	8.57	9.32
43.36M	4	4.32	10.22	10.221	10.29	10.35	10.86	11.43	12.43
71.71M	5	6.04	12.77	12.776	12.86	12.94	13.57	14.29	15.54
109.11M	6	7.94	15.33	15.332	15.43	15.53	16.29	17.15	18.64
156.45M	7	10.01	17.88	17.887	18.01	18.12	19.00	20.00	21.75
214.56M	8	12.23	20.44	20.442	20.58	20.70	21.72	22.86	24.86
284.18M	9	14.59	22.99	22.998	23.15	23.29	24.43	25.72	27.97
366.01M	10	17.09	25.54	25.553	25.72	25.88	27.14	28.58	31.07
460.72M	11	19.72	28.10	28.11	28.29	28.47	29.86	31.43	34.18
568.95M	12	22.47	30.65	30.663	30.87	31.06	32.57	34.29	37.29
691.29M	13	25.33	33.21	33.22	33.44	33.64	35.29	37.15	40.39
828.32M	14	28.31	35.76	35.77	36.01	36.23	38.00	40.01	43.50
980.61M	15	31.40	38.32	38.33	38.58	38.82	40.72	42.86	46.61
1.15G	16	34.59	40.87	40.884	41.15	41.41	43.43	45.72	49.72
1.33G	17	37.88	43.42	43.44	43.73	44.00	46.15	48.58	52.82
1.53G	18	41.27	45.98	45.99	46.30	46.58	48.86	51.44	55.93
1.75G	19	44.76	48.53	48.55	48.87	49.17	51.57	54.29	59.04

1.99G	20	48.34	51.09	51.105	51.44	51.76	54.29	57.15	62.15
2.24G	21	52.01	53.64	53.66	54.02	54.35	57.00	60.01	65.25
2.52G	22	55.77	56.20	56.22	56.59	56.94	59.72	62.87	68.36
2.81G	23	59.62	58.75	58.77	59.16	59.52	62.43	65.72	71.47
3.12G	24	63.55	61.31	61.33	61.73	62.11	65.15	68.58	74.57
3.45G	25	67.56	63.86	63.88	64.30	64.70	67.86	71.44	77.68
3.8G	26	71.65	66.41	66.44	66.88	67.29	70.58	74.30	80.79
4.17G	27	75.82	68.97	68.99	69.45	69.88	73.29	77.15	83.9
4.5G	27.82	79.30	71.06	71.09	71.56	72.00	75.52	79.50	86.44
4.57G	28	80.08	71.52	71.55	72.02	72.46	76.00	80.01	87

Now we will plot the dilation of synchronous orbit semi-axis/inner Clarke's orbit of SS in Figure 15 to Figure 21 and in Figure A.22 we give the superposition of all the curves.

Sub-satellite's Synchronous Orbit Expansion with time due to the de-spinning of Iapetus for the values of q=0,0.001,0.021,0.04,0.2,0.4 and 0.8 respectively.

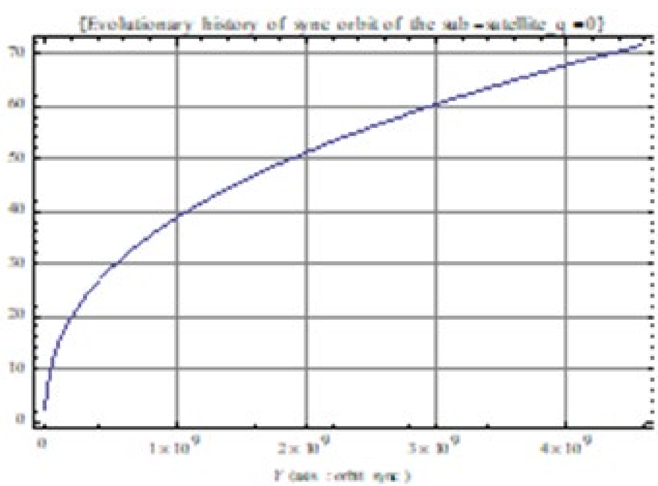

Figure A.5: Sub-satellite's Synchronous Orbit Expansion with time for q=0;

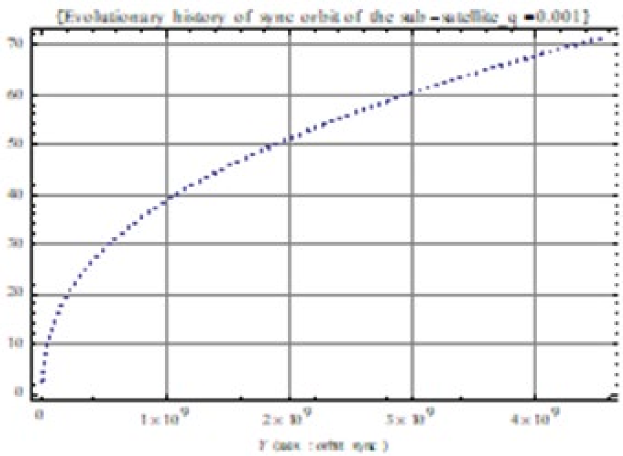

Figure A.6: Sub-satellite's Synchronous Orbit Expansion with time for q=0.001;

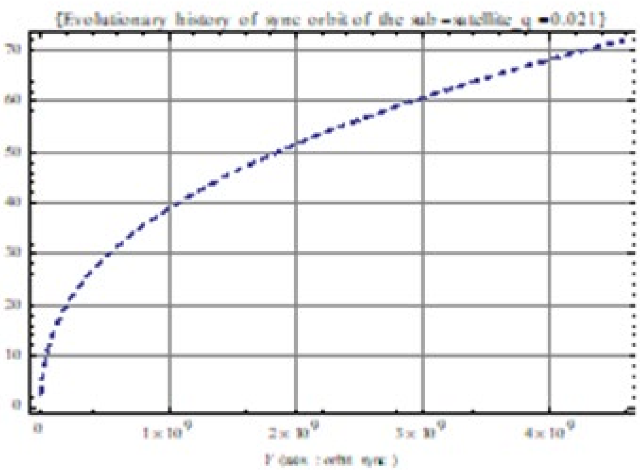

Figure A.7: Sub-satellite's Synchronous Orbit Expansion with time for q=0.021;

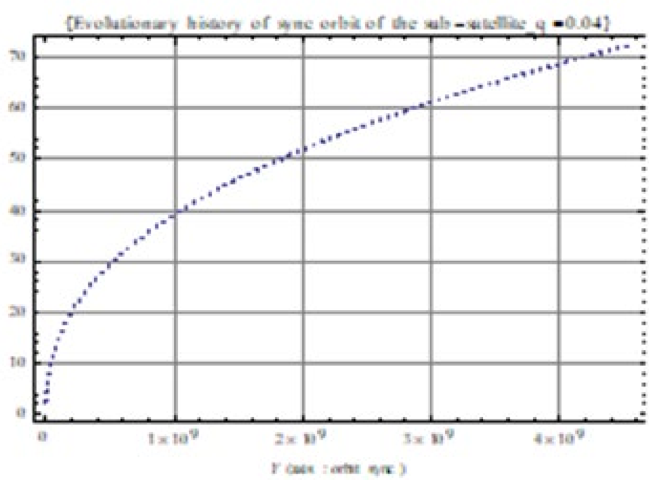

Figure A.8: Sub-satellite's Synchronous Orbit Expansion with time for q=0.04;

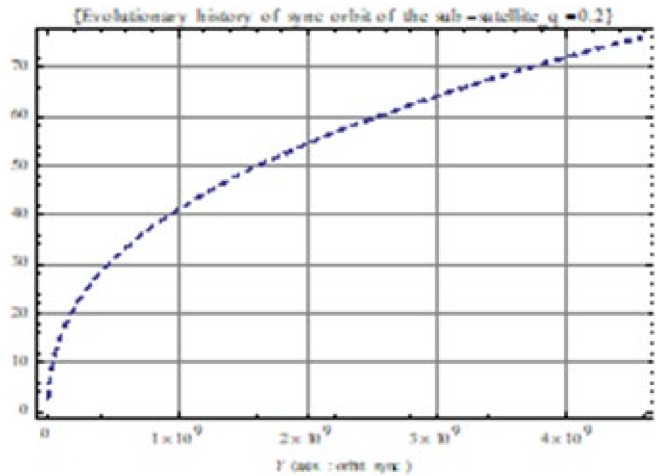

Figure A.9: Sub-satellite's Synchronous Orbit Expansion with time for q=0.2;

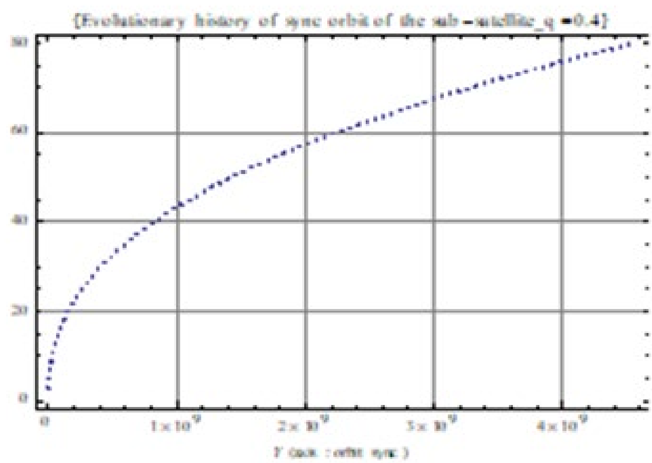

Figure A.10: Sub-satellite's Synchronous Orbit Expansion with time for q=0.4;

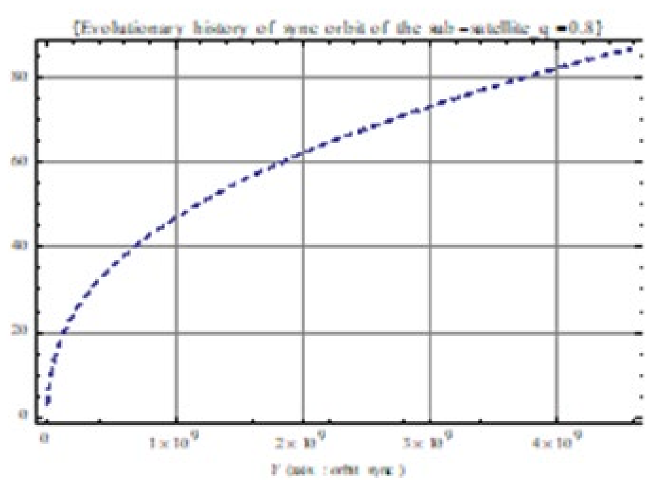

Figure A.11: Sub-satellite's Synchronous Orbit Expansion with time for q=0.8;

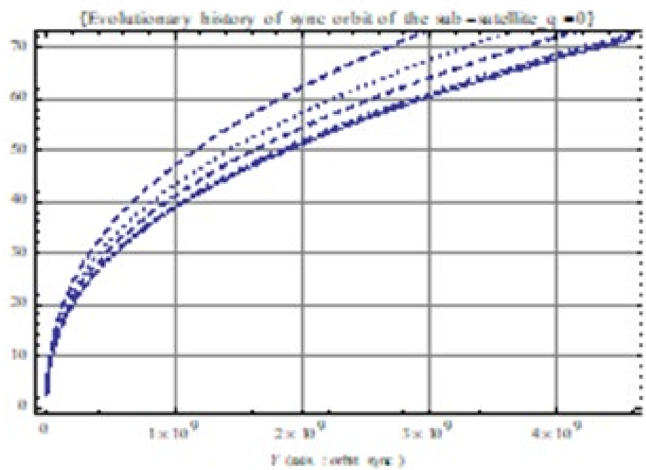

Figure A.12: Superposition of the curves in Figure A.5 to A.11. The thick curve corresponding to q=0 defines the lower limit of the range of expansion of the synchronous orbit.

Within Solar System's age (4.5Gy), minimum range of expansion of synchronous orbit is from $2.55R_{Iap}$ to $71R_{Iap}$ for q=0 and maximum range is from $3.11R_{Iap}$ to $86.44R_{Iap}$ for q=0.8.

Appendix B

The two Clarke's orbits stability and energy budget estimate of gravitational sling shot model

In order to determine the stability of the two Clarke's orbits we will have to analyze total Energy Formulation of a binary system for instance Earth-Moon System and determine its maxima and minima points.

Total energy of Earth-Moon system=Rotational kinetic energy+Potential energy+ Translational kinetic. B.1

Translational kinetic energy of the order of 1×10^{8} Joules due to recession of Moon for all practical purposes is negligible as compared to rotational kinetic energy of the order of 1×10^{30} Joules. Hence translational kinetic energy is neglected in future analysis.

Moon is trapped in potential well created by the Earth.

Moon's potential energy=$-GM_{Earth}M_{Moon}/a$

Where,

G=Gravitational constant=6.673×10^{-11} N-m^2/Kg^2;

M_{Earth}=Mass of the Earth=5.9742×10^{24} Kg;

M_{Moon}=Mass of the Moon=E/81=7.348×10^{22} Kg;

a=Semi-major axis of Moon's orbit around the Earth=3.844×10^{8} m;

Rotational kinetic energy of Earth-Moon system=Spin energy of the Earth+Orbital energy of the Earth-Moon system+Spin energy of the Moon =

$$\frac{1}{2}C\omega^2 + \frac{1}{2}\left(\frac{M_{Moon}}{1+\frac{M_{Moon}}{M_{Earth}}}\right)a^2 \times \Omega^2_{orbital} + \frac{1}{2}(0.4M_{Moon}R^2_{Moon})\Omega^2_{spin} \quad \text{B.2}$$

Where,

C=Moment of inertia around polar axis=$0.3308 M_{Earth} R_{Earth}^2 = 8.02 \times 10^{37}$ Kg-m^2;

Equatorial radius of Earth=6.37814×10^6 m;

Equatorial radius of Moon=1.738×10^6 m;

Earth angular spin velocity=$\omega = 2\pi/T_E = [2\pi/(86400)]$ radians/sec;

In this analysis we will consider all rates of rotation to be in solar days. We will consider one solar day as the present spin-period of Earth. Similarly while calculating Earth-Moon orbital angular momentum we will use present sidereal month expressed in 27.3 solar days.

Earth-Moon orbital angular velocity=$\Omega = [2\pi/(27.3 \times 86400)]$ radians/sec

Where sidereal month=27.3 d;

Since Moon is in synchronous orbit *i.e.* it is tidally locked with the Earth hence we see the same face of Moon and Moon's orbital angular velocity=Moon's spin angular velocity=Ω;

Therefore total rotational kinetic energy equation 1 reduces to:

$$\frac{1}{2}C\omega^2 + \frac{1}{2}\left(\frac{M_{Moon}}{1+\frac{M_{Moon}}{M_{Earth}}}\right)a^2 \times \Omega^2 + \frac{1}{2} \times (0.4M_{Moon}R^2_{Moon})\Omega^2 \qquad \text{B.3}$$

Similarly total angular momentum of Earth-Moon system is as follows:

$$J_T = C\omega + \left(\frac{M_{Moon}}{1+\frac{M_{Moon}}{M_{Earth}}}\right)a^2 \times \Omega + (0.4M_{Moon}R^2_{Moon})\Omega \qquad \text{B.4}$$

Substituting the numerical values in equation B.4 we obtain:

$J_T = 3.44026 \times 10^{34}$ Kg-m^2/sec;

From equation A.15, we have the following relation between length of sidereal month and length of sidereal day:

$$\frac{LOM}{LOD} = \frac{\omega}{\Omega} = E \times a^{1.5} - F \times a^2 \qquad \text{B.5}$$

$$Where\ E = \frac{J_T}{BC}\ and\ F = \left(\frac{M_{Moon}}{C(1+\frac{M_{Moon}}{M_{Earth}})}\right)$$

$$Here\ B = \sqrt{G(E+m)}$$

Substituting the numerical values we get:

$B=20.08884482 \times 10^{6}\ m^{3/2}/s$;

$E=2.13531 \times 10^{-11}\ m^{-3/2}$;

$F=9.05036 \times 10^{-16}\ m^{-2}$;

If the numerical values of E and F are substituted in equation B.5 and the present value of 'a' is substituted we get LOM/LOD=27.2 we should get 27.3. This is because equation A.5 has been derived based on Keplarian Approximation. If LOM/LOD was derived from exact analysis we would get LOM/LOD in the present epoch as 27.3.

$$\frac{LOM}{LOD} = \frac{\omega}{\Omega} = E \times a^{1.5} - F \times a^{2} = 1 \qquad \text{B.6}$$

Equation B.6 defines geosynchronous orbits of Earth-Moon system when both are tidally interlocked and are in triple synchrony state:

$T_{orbit}=T_{spinmoon}=T_{spinearth}$

If equation B.6 is solved we get two roots:

$a_{G1}=1.46177 \times 10^{7}$ m and $a_{G2}=5.52656 \times 10^{8}$ m;

If expressed as the ratio a/R_{Earth} we get:

$a_{G1}=2.29$ and $a_{G2}=86.65$;

Rewriting total rotational Kinetic Energy expression from equation B.3 we get:

$$KE = \frac{1}{2}C\omega^{2} + \frac{1}{2}\left(\frac{M_{Moon}}{1+\frac{M_{Moon}}{M_{Earth}}}\right)a^{2} \times \Omega^{2} + \frac{1}{2} \times (0.4M_{Moon}R_{Moon}^{2})\Omega^{2}$$

Reshuffling the angular velocity terms we get:

$$KE = \frac{1}{2}\Omega^{2}\left[C\left(\frac{\omega}{\Omega}\right)^{2} + \left(\frac{M_{Moon}}{1+\frac{M_{Moon}}{M_{Earth}}}\right)a^{2} + (0.4M_{Moon}R_{Moon}^{2})\right]; \qquad \text{B.7}$$

Substituting equation B.6 in equation B.7 we get:

$$KE = \frac{1}{2}\Omega^{2}\left[C(E \times a^{1.5} - F \times a^{2})^{2} + \left(\frac{M_{Moon}}{1+\frac{M_{Moon}}{M_{Earth}}}\right)a^{2} + (0.4M_{Moon}R_{Moon}^{2})\right]; \qquad \text{B.8}$$

According to Kepler's 3rd Law:

$$a^3\Omega^2 = G(M_{Earth} + M_{Moon}) \qquad \text{B.9}$$

Substituting equation B.9 in equation B.8 we obtain:

$$KE = \frac{1}{2} \times \frac{G(M_{Earth} + M_{Moon})}{a^3}\left[C(E \times a^{1.5} - F \times a^2)^2 + \left(\frac{M_{Moon}}{1 + \frac{M_{Moon}}{M_{Earth}}}\right)a^2 + (0.4 M_{Moon} R_{Moon}^2)\right]; \qquad \text{B.10}$$

Therefore total energy of the E-M System is:

$$TE = KE + PE$$

Therefore:

$$TE = \frac{1}{2} \times \frac{G(M_{Earth}+M_{Moon})}{a^3}\left[C(E \times a^{1.5} - F \times a^2)^2 + \left(\frac{M_{Moon}}{1+\frac{M_{Moon}}{M_{Earth}}}\right)a^2 + (0.4 M_{Moon} R_{Moon}^2)\right] - \frac{G M_{Earth} M_{Moon}}{a} \qquad \text{B.11}$$

To determine the stable and unstable equilibrium points in non-Keplerian journey of Moon we must examine the Plot of equation B.11 from 'a'=8×10^6 to 'a'=6×10^8 m;

$0.5 \times (G/a)(((m + M)/a^2)(C \times (E \times a^{1.5} - F \times a^2)^2 + (m/(1 + 1/81)) \times a^2 + 0.4 \times m \times R^2) - 2 \times m \times M)/.\{G \to 6.673 \times 10^{-11}, m \to 7.348 \times 10^{22}, M \to 5.9742 \times 10^{24}, C \to 8.02 \times 10^{37}, R \to 1.738 \times 10^6, E \to 2.1353127743727534 \times 10^{-11}, F \to 9.050361900127731 \times 10^{-16}\}$

B.12

$\frac{1}{a} 3.3365 \times 10^{-11}(-8.77968432 \times 10^{47} + \frac{1}{a^2} 6.047679999999999 \times 10^{24}(8.8782768448 \times 10^{34} + 7.258390243902439 \times 10^{22} a^2 + 8.02 \times 10^{37}(2.135312774372753 \times 10^{-11} a^{1.5} - 9.050361900127731 \times 10^{-16} a^2)^2))$ B.13

Plot $\left[\frac{1}{a} 3.3365 \times 10^{-11}\left(-8.77968432 \times 10^{47} + \frac{1}{a^2} 6.047679999999999 \times 10^{24}(8.8782768448 \times 10^{34} + 7.258390243902439 \times 10^{22} a^2 + 8.02 \times 10^{37}(2.135312774372753 \times 10^{-11} a^{1.5} - 9.050361900127731 \times 10^{-16} a^2)^2)\right), \{a, 1.4 \times 10^7, 1.5 \times 10^7\}, \text{GridLines} \to \text{Automatic}, \text{Frame} \to\right.$

$\text{True}, \text{FrameLabel} \to \text{semi} - \text{majoraxis}(a)\text{meter}, \text{PlotLabel} \to \{\text{Total Energy Plot}\}\Big]$
B.14

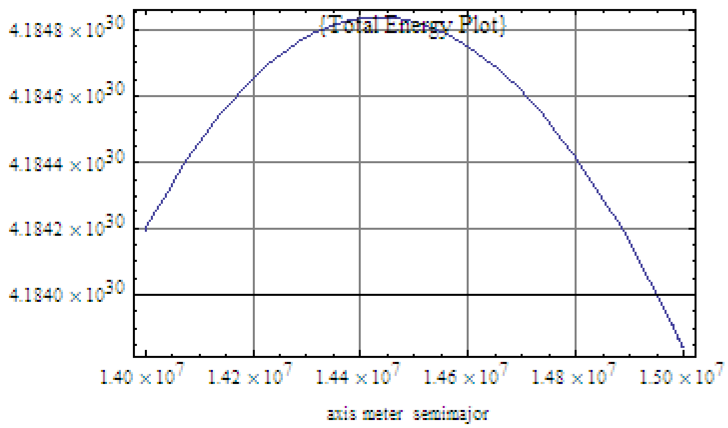

Figure B.1: Plot of total energy in the range 1.4×10^7 m to 1.5×10^7 m around the inner geo-synchronous orbit of a=1.46×10^7 m. X-axis–m; Y-axis -Joules

We find energy Maxima at inner geo-synchronous orbit hence it is unstable equilibrium point. When Moon is at inner-geosynchronous orbit, any perturbation launches Moon on either a sub-synchronous orbit or on extra-synchronous (or super-synchronous orbit). If it is launched on sub-synchronous orbit then it rapidly spirals in towards the primary body and if it is launched on extra-synchronous orbit then it spirals out from inner to outer geosynchronous orbit. In our case, Moon is fully formed beyond Roches' Limit which is 18,000 Km just beyond inner Clarke's orbit or inner Geo-synchronous Orbit hence Moon is launched on expansionary spiral orbit towards outer Clarke's Orbit or outer Geo-synchronous Orbit.

$\text{Plot}[\frac{1}{a}3.3365 \times 10^{-11}(-8.77968432 \times 10^{47} + \frac{1}{a^2}6.047679999999999 \times 10^{24}(8.8782768448 \times 10^{34} + 7.258390243902439 \times 10^{22}a^2 + 8.02 \times 10^{37}(2.135312774372753 \times 10^{-11}a^{1.5} - 9.050361900127731 \times 10^{-16}a^2)^2)), \{a, 5.4 \times 10^8, 5.6 \times 10^8\}, \text{GridLines} \to \text{Automatic}, \text{Frame} \to \text{True}, \text{FrameLabel} \to \text{semimajoraxismeter}, \text{PlotLabel} \to \{\text{"Total Energy Plot"}\}]$
B.15

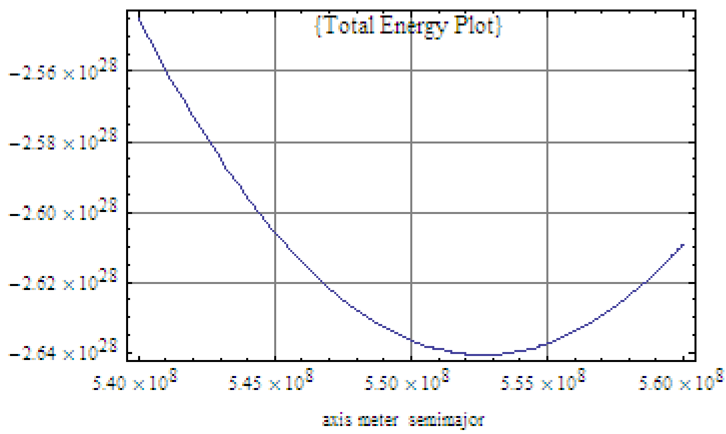

Figure B.2: Plot of total energy in the range 5.4×10^8 m to 5.6×10^8 m around the outer geo-synchronous orbit of a=5.527×10^8 m.

At outer geosynchronous orbit there is energy minima hence it is stable equilibrium point. Secondary body can never move beyond this orbit. Either it is stay-put in that orbit or it gets deflected back into a contracting spiral orbit.

Plot[(1/a)3.3365`*^-11 (-8.779684319999999`*^47+1/a^26.047679999999999`*^24 (8.8782768448`*^34+7.258720964072173`*^22 a^2+8.019999999999999`*^37 (2.1353128`*^-11 a^1.5`-9.050361900000001`*^-16 a^2)^2)),{a,0.5` 10^7,8` 10^8},PlotStyle->Thick,GridLines->Automatic,Frame->True,FrameLabel->a meter,PlotLabel->{Total Energy Plot}]

B.16

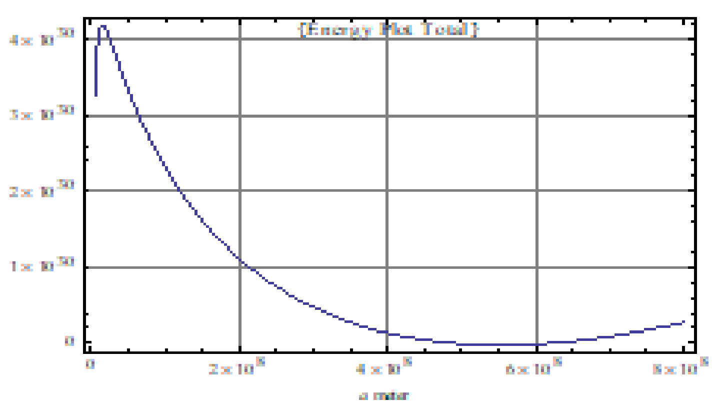

Figure B.3: Plot of total energy in the range 0.5×10^7 m and 8×10^8 m along the long tidal history of Moon from its inception to its lock-in at outer geo-synchronous orbit.

In a similar work by G.A.Krasinsky the results obtained are as follows:

"Analytical consideration show that if the contemporary lunar orbit were equatorial the evolution would develop from an unstable geosynchronous orbit of the period 4.42 h (in the past) to a stable geosynchronous orbit of the period 44.8 days (in the future). It is also demonstrated that at the contemporary epoch the orbital plane of the fictitious equatorial moon would be unstable in the Liapunov's sense, being asymptotically stable at the earlier stages of the evolution."

In Table B.1, a comparison of results of my analysis and that of Krasinsky is given.

Table B.1: Comparative study of the results obtained by Krasinsky, Sharma (personal communication:arXiv:0805.0100 (2008)); and Darwin.

	a_{G1}/R_{Earth}	a_{G2}/R_{Earth}	Orbital period at aG1	Orbital period at aG2
Krasinsky analysis	2.15	83.8	4.42 h	44.8 days
BKS analysis	2.29	86.65	4.8596 h	47.0739 days
Darwin Analysis	Not available	90.4	Not available	47 days

B.1. Energy budget of gravitational sling–shot model of Earth-Moon system

The basic physics of gravitational sling-shot model is that when there is considerable differential between Earth's spin velocity (ω) and Moon's orbital velocity (Ω) there is oscillatory changes in the tidal deformation of Earth and Moon. The tidally deformed shape oscillates between extreme oblateness (and stretching) to extreme prolateness (or squeezing). It is this rapid oscillation between the two extremes which leads to dissipation of energy and tidal heating because of anelastic nature of Earth as well as Moon. This heating takes away energy from Moon's rotation. Moon's rotation slows down until its spin and orbital period are the same. Once spin and orbital period are synchronized energy loss from Moon stops.

Tidal locking time scale of the secondary (depends on the size of the orbit and the mass of the parent star) is $=10^{12}\, years \times (\frac{a}{1AU})^6 \times (\frac{M}{M_{Sun}})^{-2}$ B. 17

Tidal locking Radius $=0.4AU \times (\frac{M}{M_{Sun}})^{1/3}$ $B.18$

In case of Earth and Moon, Earth will be treated as the host or the primary.

But at the geo-synchronous orbits ω=Ω and Earth and Moon are tidally interlocked and during the lock-in stage the two bodies are permanently deformed in oblate shape therefore at these two points we have the conservation of energy and the system is a conservative system.

On the basis of the above reasoning, we have assumed the following:

- From a_{G1} to a_2, Earth-Moon System is a conservative system and Moon experiences a powerful sling-shot impulsive torque;

- From a_2 to a_{G2}, Earth-Moon System is a dissipative system;
- From a_{G1} to 0, Earth-Moon System is a dissipative system;

During the conservative phase from a_{G1} to a_2 when the secondary tumbles in super-synchronous orbit its tidal drag slows down the spin of Earth and pushes out Moon in an expanding spiral orbit as shown Figure A.1 thereby increasing the orbital period of the Moon and increasing the PE of the E-M system.

At the time Moon is spiraling out three things are happening:

- The reduction in spin and orbital energy is partially transferred to increase the PE of the system;
- Partially transferred to the translational KE of Moon;
- Remaining is dissipated as heat.

Hence we can say that:

K×(reduction in Earth Spin Energy +reduction in orbital energy)–increase in Potential Energy=translational Kinetic Energy in the radial direction. B.19

$2\pi^2\times C\ [1/(P1)^2 - 1/(P2)^2]/.\ C\rightarrow 8.02\times 10^{37}$ B.20

$1.583084545934732\times 10^{39}\left(\frac{1}{P1^2}-\frac{1}{P2^2}\right)$ B. 21

1.583084545934733`*^39 $(1/P1^2-1/P2^2)/.\{P1\rightarrow 0.2023\times 86400,\ P2\rightarrow 0.214067\times 86400\}$ B.22

Spin rotational energy given up by Earth=5.54023*10^29 Joules. B.23

P1 and P2 are the spin period of the Earth when the system are in a_{G1} and a_2 configuration.

At $a_{G1}=1.46177\times 10^7$ m, the Orbital Period of Moon is P1=0.2023 d (4.9 hr) by Kepler's Third Law.

At $a_2=2.40942\times 10^7$ m, the Orbital Period of Moon is P2=0.4218 d (10.1 hr). This is 2:1 Mean Motion Resonance Point.

While Moon's orbital period increases from 0.2023 d (4.9 hr) to 0.4218 d (10.1 hr). Since a_2 is 2:1 MMR position hence Earth's spin period is half of 0.4218 d. Therefore Earth's spin period increases from 0.2023 d (4.9 hr) to 0.214067d (5.14 hr).

Both these slowing down and de-spinning lead to the transfer of rotational KE from the Earth and Moon to the Earth-Moon system. This transfer increases the PE of the system and simultaneously imparts the maximum radial velocity to our Moon. It is this recessionary radial velocity which enables our Moon to coast on its own from a_2 to a_{G2} orbit. At a_{G2}, radial velocity becomes zero and the spiral path terminates at a_{G2} orbital radius. Our Moon will be deflected back into collapsing spiral path due to Sun's tidal interaction.

$$2\pi^2(I + m^* \times x_1^2) \times \frac{1}{P_1^2} - 2\pi^2(I + m^* \times x_2^2) \times \frac{1}{P_2^2} \ /. \{m^* \to 7.258721 \times 10^{22}, I \to 8.878277 \times 10^{34}\} \qquad \text{B.24}$$

In the above equation, m*=M_{Moon}/(1+M_{Moon}/M_{Earth})=reduced mass of our Moon=7.258721 × 10^{22} Kg.

I=Moment of inertia of our Moon around its spin axis=0.4M_{Moon} R_{moon}^2=

8.878277 × 10^{34} Kg-m^2.

The parameters x1 and x2 are a_{G1} and a_2 and P1 and P2 are the orbital period of Moon 0.2023d and 0.4281d at a_{G1} and a_2.

2 × [Pi]^2 ((8.87828*10^34+7.25872*10^22x1^2)/P1^2(8.87828*10^34+7.25872*10^22 x2^2)/P2^2 B.25

2\[Pi]^2 ((1/(P1^2))(8.878277000000001`*^34+7.258721000000001`*^22 x1^2)-1/P2^2 (8.878277000000001`*^34+7.258721000000001`*^22 x2^2))/.{x1->1.46177*10^7,x2->2.40942*10^7} B.26

2 (1.5599*10^37/P1^2-4.22279*10^37/P2^2) \[Pi]^2 B.27

2 (1.5599011168063421`*^37/P1^2-4.2227870171506146`*^37/P2^2) \[Pi]^2/.{P1->0.2023*86400,P2->0.4281*86400} B.28

Orbital energy given up by E-M system=3.986053971911033 × 10^{29} Joules. B.27

Gain of PE=G*E*m(1/x1-1/x2)/.{G->6.673*10^-11,E->5.9742*10^24,m->7.348*10^22} B.29

2.92934*10^37 (1/x1-1/x2) B.30

2.9293416733679992`*^37(1/x1-1/x2)/.{x1->1.46177*10^7,x2->2.40942*10^7} B.31

$$\textbf{Increase in the PE of E} - \textbf{M system} = \mathbf{7.88181870890782} \times \mathbf{10^{29}}\textbf{Joules} \qquad \text{B.31}$$

If all the rotational energy given up by Earth and Moon are conserved during conservative phase then:

Rotational energy given up by Earth+Orbital energy given up by E-M system=Increase in the PE of E − M system + increase in translational KE in radial direction B. 32

Solving equation B.32 we get:

5.540225834158889`*^29+3.986053971911033`*^29-7.88181870890782`*^29= $1.644461097162102 \times 10^{29} = 0.5 \times m \times v_{max}^2$ B.33

Solving equation B.33 we get the solution for v_{max}:

Sqrt[2*1.644461097162102`*^29/m]/.m->7.348*10^22 B.34

v_{max}=2115.64 m/sec B.35

By Lunar Laser Ranging present velocity of recession is 3.8 cm per solar year and at MMR 2:1 from primary-centric analysis we have v_{max}=1.7178 m/solar year=5.4435×10^{-8} m/s.

This velocity tells us that increase in translational KE in radial direction=1.08867×10^{8} Joules. B.36

Solve[K(5.540225834158889`*^29+3.986053971911033`*^29)-7.88181870890782`*^29==1.08867*10^8,K] B.37

{{K->0.827376}} B.38

Equation B.38 tells us that 82.7376% of spin and orbital energy given by Earth, Moon and E-M system are transferred to the increase in PE and translational KE and remaining part of the rotational energy is lost as heat.

$$TE = \frac{1}{2} \times \frac{G(m+M)}{a^3}\left[C(E \times a^{1.5} - F \times a^2)^2 + \left(\frac{m}{1+\frac{m}{M}}\right)a^2 + (0.4mR_{Moon}^2)\right] - \frac{GMm}{a} \qquad \text{B. 39}$$

When equation (B.39) is solved:

At maxima *i.e.* at a_{G1}, TE=4.18477×10^{30} J. B.40

At minima *i.e.* at a_{G2}, TE=-2.64045×10^{28} J. B.41

TE cuts the zero axis at 4.87914×10^{8} m and at 6.2136×10^{8} m. B.42

When our Moon recedes from 2:1MMR position *i.e.* from $a_2=2.40942 \times 10^7$ m to $a=4.87914 \times 10^8$ m:

- Earth's spin energy released is 4.59781×10^{30} J by solving equation B.18 while Earth de-spins from 0.214067 d (5.14 hrs) to 2.658 d because at $a=4.87914 \times 10^8$ m, LOM/LOD=14.6785 and LOM=39.0145 d therefore LOD=2.658 d;
- E-M orbital energy and Moon's spin energy released is 5.79252×10^{29} J by solving equation B.22 while Moon spin and orbital period de-spins from 0.4218d (10.1 hrs) to 39.0145d ;
- Gain in PE=1.15575×10^{30} J;

Following is the energy budget equation:

Solve[K*4.59781*10^30+5.79252*10^29=1.15575*10^30,K] B.43

{{K->0.125385}} B.44

This means only 12.5% of Earth's spin energy is transferred to E-M system. 88.5% of energy released goes for tidal heating of our Earth during its journey from a_2 to $a=4.87914 \times 10^8$m. This is an expected result. Total Energy of the system is plotted in Figure B.4

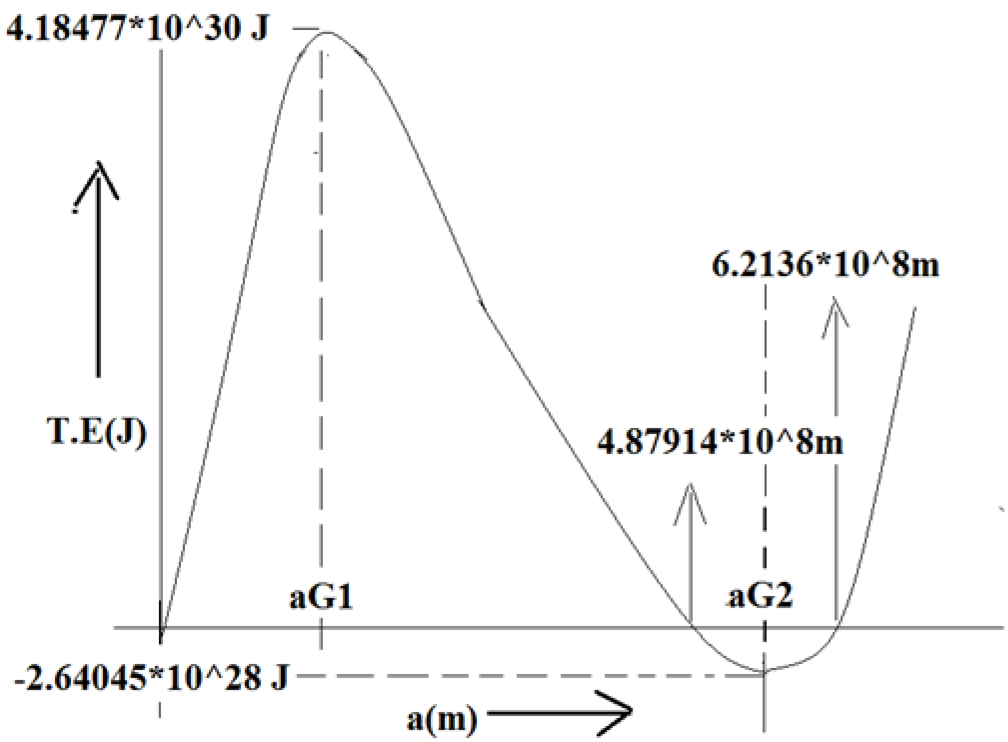

Figure B.4: Total energy plot of E-M system with respect to 'a'm.

Appendix C

Kinematic Model based analysis of the hypothetical Sub-satellite

C.1. Calculation of the two Clarke's Orbits for the impact-generated sub-satellite for different mass ratios from q=0.0001 to q=1.0

To calculate the two Clarke's orbits, total angular momentum (J_T) of sub-satellite(SS)-Iaptus system, the rotational inertia or the moment of inertia (C_{Iap}) of Iapetus around its spin axis and the parameter $B=\sqrt{(G(M_{Iap}+M_{SS}))}$ are calculated. From these three parameters (J_T, C_0 and B) the constants E and F of 'spin to orbital angular velocity equation' are calculated where E and F are:

$$E = \frac{J_T}{B^* C_{Iap}} \; and \; F = \frac{M_{SS}}{1+\frac{M_{SS}}{M_{Iap}}} \times \frac{1}{C_{Iap}} \qquad \text{C. 1}$$

The 'spin to orbital angular velocity equation' is set up:

$$E \times a^{1.5} - F \times a^2 = \frac{\omega}{\Omega} \qquad \text{C. 2}$$

Equating this equation to Unity yields two roots a_{G1} and a_{G2} which are called the inner and outer Clarke's orbit.

SS is at unstable equilibrium at inner Clarke's orbit hence it tumbles out of the orbit (Appendix B). If it falls short of a_{G1} it is in sub-synchronous orbit and it spins-up the binary system and itself spirals-in to its certain merger with Iapetus.

If it falls long of a_{G1} SS is in super-synchronous orbit and it de-spins the binary system and SS itself spirals out to a_{G2}.

In Table C.1. All the kinematic parameters, constants of equation and the two Clarke orbits semi-major axes are tabulated for different mass ratios.

$M_{Iap}=1.8 \times 10^{21}$ Kg, $R_{Iap}=7.3563 \times 10^5$ m and the synchronous orbits (given by Equation 8 in the main text) of SS for q=0.0001 to 0.1 to 1 is taken as given in the Table C.1.

I assume that SS is launched on an expanding spiral orbit from the Triple Synchrony orbit which is a_{synSS} as defined by Levison et al.

C_{Iap}=The principal moment of inertia of Iapetus around Polar Axis=0.4 $M_{Iap}R_{Iap}^2$.

Substituting the numerical values, $C_{Iap}=3.896290778 \times 10^{32}$ Kg-m^2.

Equation C.3 gives the total angular momentum at the time of gravitational sling shot launching of SS just beyond inner Clarke's orbit. At the time of launching triple synchrony of 12.971 hours is assumed at the inner Clarke's orbit as discussed in section 3 of main text. In this particular calculation the inner Clarke's orbits are not known hence the orbit of SS around Iapetus which corresponds to 12.971 hours as the orbit in which SS is fully formed by accretion and this will be our starting point in our calculations. As we will see subsequently that 'a_{synSS}' is inner Clarke's Orbit for q~0 to q=0.006. Again at q approaching 1, 'a_{synSS}' is approaching outer Clarke's Orbit but at q=1, 'a_{synSS}' falls short of a_{G2}. So Primary-centric analysis is a valid theoretical formulation.

We assume that:

$P_{spinSS}=P_{spinIap}=P_{orbitSS}=12.971$ hours.

Since Iapetus is formed just beyond a_{Roche} hence we assume that Iapetus is fully formed and placed at $a_{Iapi}=1.28 \times 10^8$ m with a spin period=orbital period=12.971 hours since Iapetus is in captured rotation.

We assume that SS is formed for a given 'q' at its respective triple synchrony Orbit. Hence for each value of 'q', we assume that Iapetus-Subsatellite system is in mutually tidally interlocked state hence $P_{spinSS}=P_{spinIap}=P_{orbitSS}=12.971$ hours.

We assume 'a_{SS} to be semi-major axis of the orbit in which SS is in an orbital period=spin period=12.971 hours. Since Iapetus has a spin period of 12.971 hours by virtue of captured rotation hence it is reasonable to assume that after SS is fully formed it is in triple synchrony. This is permissible since Roche's Limit for SS is $2.495R_{Iap}=1.83539685 \times 10^6$ m (irrespective of M_{SS}) and the orbits of SS corresponding to 12.971 hours orbital period range from 1.879×10^6 m(=2.5544 R_{Iap}) to 2.3674×10^6 m(=3.218 R_{Iap}) for 'q' ranging from '0' to '1'. So for each value of 'q', physically it is possible that SS was born in triple synchrony orbit.

$$J_T = 0.4M_{Iap}R_{Iap}^2 \times \left(\frac{2\pi}{P_{spinIap}}\right) + 0.4M_{SS}R_{SS}^2 \times \left(\frac{2\pi}{P_{spinSS}}\right) + \frac{M_{SS}}{1+\frac{M_{SS}}{M_{Iap}}} a_{SS}^2 \times \left(\frac{2\pi}{P_{orbitSS}}\right) \qquad \text{C.3}$$

$$where\ a_{SS} = \left(\frac{B^*}{\left(\frac{2\pi}{P_{orbitSS}}\right)^2}\right)^{1/3} \quad and\ B^* = \sqrt{G(M_{Iap}+M_{SS})}\ and\ P_{spinIap} = P_{spinSS} = P_{orbitSS}$$

Table C.1: The physical parameters, the kinematic parameters, the constants of equation for different mass ratios ranging from q=0.0001 to q=1.

q	M_{SS}($\times 10^{17}$ Kg)	R_{SS}(m)†	a_{synSS} ($\times R_{Iap}$)	$B^* m^{3/2}/sec$	J_T($\times 10^{29}$) Kg-m^2/s	E($\times 10^{-10}$) m$^{-1.5}$	F($\times 10^{-14}$) m^{-2}
0.0001	1.8	35026.33	2.56	346592	0.525085	3.8883	0.046193
0.001	18	75461.94	2.56	346747.9	0.532807	3.94371	0.461516
0.006	108	137123.4	2.56	347612.8	0.575695	4.25055	2.75534
0.009	162	156967.16	2.57	348130.76	0.601404	4.443376	4.12071
0.021	378	208193.77	2.58	350194.8	0.703979	5.1594	9.50199
0.03	540	234477.8	2.58	351734.87	0.780608	5.69595	13.4557

0.04	720	258076.2	2.59	353438.2	0.865426	6.28442	17.7684
0.05	900	278004.2	2.60	355133.36	0.949895	6.846488	21.9987
0.06	1080	295423.51	2.61	356820.46	1.03401	7.43745	26.1497
0.07	1260	311000.18	2.62	358499.62	1.11778	8.0023	30.2228
0.08	1440	325155.64	2.62	360170.96	1.20119	8.55956	34.2206
0.09	1620	338175.50	2.63	361834.58	1.28426	9.1094	38.145
0.1	1800	350263.3	2.64	363490.58	1.36698	9.65198	41.998
0.2	3600	441304	2.72	379654	2.17618	14.7115	76.9963
0.3	5400	505167	2.79	395156	2.95598	19.1991	106.610
0.4	7200	556008	2.86	410073	3.71077	23.2248	131.994
0.5	9000	598941.8	2.93	424465.55	4.44427	26.8724	153.993
0.6	10800	636471	2.99	438386	5.15955	30.2068	173.242
0.7	12600	670030	3.05	451878	5.85915	33.2784	190.226
0.8	14400	700526.6	3.11	464978.71	6.54518	36.1274	205.323
0.9	16200	728577	3.16	477720	7.21941	38.7862	218.832
1.00	18000	754619	3.22	490131	7.88335	41.2807	230.989

Note: $*B=\sqrt{[G(M_{Iap}+M_{SS})]}$; $^{\dagger}R_{SS}$ radius of SS is determined by assuming a density of Sub-satellite=1000Kg/m³;

I have assumed that SS is launched in an orbit of triple synchrony state of period 12.971 hours. Theoretically it was expected that Kinematic Model analysis will yield inner Clarke's Orbit $a_{G1}=a_{synSS}$ but things donot turn up quite like that.

Table C.2: The two Clarke's orbits semi-major axes and mean motion resonance (ω/Ω=2) semi-major axes for q=0.0001 to q=1.

Col.1	**Col.2**	**Col.3**	**Col.4**	**Col.5**	**Col.6**	**Col.7**	**Col.8**	**Col.9**
q	a_{synSS} (×R_{Ia})	a_{G1}(×10^6)m	a_{G1}(×R_{Ia})	MMR 2:1(×R_{Ia})	a_{G2}(×10^6)m	a_{G2}(×R_{Ia})	E†	F†

0.0001	2.5545	1.8792	2.5546	4.056	708545	963221	3.88	0.046
0.001	2.5553	1.8797	2.5553	4.068	7301.9	9926.4	3.94	0.462
0.006	2.56	1.8822	2.559	4.135	237.67	323.1	4.25	2.755
0.009	2.562	1.8836	2.561	4.179	115.35	156.81	4.443	4.121
0.021	2.572	1.8884	2.567	4.38	28.746	39.078	5.159	9.502
0.03	2.58	1.891	2.571	4.59	17.036	23.159	5.695	13.46
0.04	2.588	1.8929	2.573	4.92	11.511	15.65	6.284	17.77
0.05	2.596	1.8936	2.574	5.446	8.6559	11.77	6.865	22
0.06*	2.604	1.893	2.573	4.04 (1.5)	6.9443	9.44	7.437	26.15
0.07*	2.613	1.8907	2.57	4.44 (1.5)	5.8176	7.91	8.002	30.22
0.08*	2.621	1.8865	2.565	4.41 (1.4)	5.0265	6.83	8.556	34.22
0.09*	2.629	1.8801	2.556	4.07 (1.3)	4.4453	6.04	9.109	38.15
0.1*	2.637	1.8711	2.544	3.47 (1.2)	4.0041	5.44	9.652	42
0.2*	2.714	1.5811	2.149	2.69 (1.08)	2.5081	3.41	14.71	77
0.3*	2.788	1.2194	1.658	1.93 (1.1)	2.3964	3.26	19.2	106.6
0.4*	2.858	0.997	1.355	2.05 (1.3)	2.4335	3.308	23.22	132
0.5*	2.924	0.8561	1.164	2.21 (1.5)	2.4983	3.396	26.87	154
0.6*	2.988	0.7595	1.032	1.88 (1.6)	2.5717	3.496	30.21	173.2
0.7*	3.049	0.6889	0.936	1.94 (1.8)	2.6486	3.601	33.28	190.2
0.8	3.107	0.6349	0.863	1.99	2.7271	3.707	36.13	205.3
0.9	3.164	0.5921	0.805	1.6976	2.8064	3.815	38.79	218.8
1	3.218	0.5573	0.758	1.527	2.886	3.92	41.28	231

Note: †E(×10^{-10})$m^{-1.5}$, F(×10^{-14})m^{-2}, *For the mass ratios q=0.06 to 0.7, the system does not yield REAL MMR(2:1) semi-major axes. Therefore the ratio (ω/Ω) as shown in the bracket is taken as the point where radial velocity=v_{max} is achieved by Gravitational Sling Shot impulsive torque.

Table C.3: The comparative study of inner Clarke's Orbit and the sum of Iapetus and SS radii for different mass ratios ranging from q=0.0001 to q=1 and its implication for the formation process.

q	a_{synSS} (×R_{Ia})	a_{G1} (×10^6) m	a_{G1} (×R_{Ia})	R_{SS}(m)†	(R_{Iap}+ R_{SS}) (m)	(R_{Iap}+ R_{SS}) (×R_{Ia})	Formation process
0.0001	2.5545	1.8792	2.5546	35026	770656	1.05	Core accretion
0.001	2.5553	1.8797	2.5553	75462	811092	1.1	Core accretion
0.006	2.56	1.8822	2.559	137123	872753	1.19	Core accretion
0.009	2.562	1.8836	2.561	156967	892597	1.21	Core accretion
0.021	2.572	1.8884	2.567	208194	943824	1.28	Core accretion
0.03	2.58	1.891	2.571	234478	970108	1.32	Core accretion
0.04	2.588	1.8929	2.573	258076	993706	1.35	Core accretion
0.05	2.596	1.8936	2.574	278004	1E+06	1.38	Core accretion
0.06*	2.604	1.893	2.573	295424	1E+06	1.4	Core accretion
0.07*	2.613	1.8907	2.57	311000	1E+06	1.42	Core accretion

0.08*	2.621	1.8865	2.565	325156	1E+06	1.44	Core accretion
0.09*	2.629	1.8801	2.556	338176	1E+06	1.46	Core accretion
0.1*	2.637	1.8711	2.544	350263	1E+06	1.48	Core accretion
0.2*	2.714	1.5811	2.149	441304	1E+06	1.6	Core accretion
0.3*	2.788	1.2194	1.658	505167	1E+06	1.69	Instability
0.4*	2.858	0.997	1.355	556008	1E+06	1.76	Instability
0.5*	2.924	0.8561	1.164	598942	1E+06	1.81	Instability
0.6*	2.988	0.7595	1.032	636471	1E+06	1.86	Instability
0.7*	3.049	0.6889	0.936	670030	1E+06	1.91	Instability
0.8	3.107	0.6349	0.863	700527	1E+06	1.95	Instability
0.9	3.164	0.5921	0.805	728577	1E+06	1.99	Instability
1	3.218	0.5573	0.758	754619	1E+06	2.02	Instability

In Iapetus-Sub-Satellite system,

Roche's Limit=a_{Roche_SS}=2.43 $R_{Iap}(\rho_{Iap}/\rho_{SS})^{1/3}$=2.495 R_{Iap} taking ρ_{SS}=1000 Kg/m and ρ_{Iap}=1083 Kg/m^3.

Inspection of the Table C.2 and Table C.3 show that for mass ratios q=0.0001 to 0.006, inner Clarke's Orbit (a_{G1})=a_{synSS} and a_{G1}>(R_{Ia}+R_{SS}). This means that SS has formed by normal core-accretion process. Since a_{G1}>a_{Roche_SS} hence SS is formed at a_{G1} and launched on super-synchronous orbit.

From q=0.007 to 0.2, a_{G1}>(R_{Ia}+R_{SS}) hence core accretion process is the legitimate pathway for the formation of SS but in each of these cases a_{synSS}>a_{G1} and a_{synSS}>a_{Roche_SS}hence SS in these cases are doomed to death spiral right from the beginning. This means SS in this region q=0.007 to 0.2 donot contribute to de-spinning and they in most cases contribute to a recent equatorial ridge.

Whereas for mass ratios 0.21 to 1.0, inner Clarke's Orbit (a_{G1}) is much less than a_{synSS} and a_{G1}<(R_{Ia}+R_{SS}). This means the binary components have been formed by hydro-dynamic instability.

Roche's limit criteria does not apply. In the whole range from q=0.21 to q=1.0 there is no evolutionary history. By hydrodynamic instability the two components are formed and they, on a time scale of months/years acquire the stable configuration corresponding to the outer Clarke's orbit as is the case in Pulsar-Star, Brown-Dwarf pair or Star +Brown-Dwarf pair. The spin of the primary=the spin of the secondary=the orbital period of the binary are at a steady state value There is neither de-spinning nor spin-up.

So we can divide the mass ratio q=0.0001 to 1.0 in three zones:

- From q=1.0 to 0.21 is the zone where SS gets locked at outer Clarke's Orbit instantaneously. The outer Clarke's Orbit is at=3.88R_{Iap} for q=1.0 which corresponds to 17.2 hours. The outer Clarke's Orbit is at 3.2R_{Iap} for q=0.3 which corresponds to 15.95 hours. At q=0.3, 'Iapetus+SS' immediately assume a stable configuration with orbital period of SS=spin period of SS=spin period of Iapetus~16 hours.
- From q=0.2 to 0.007, the SS gets trapped in death spiral right from the beginning. In this zone SS is destined to spiral-in and impact Iapetus. It does not contribute to de-spinning and does not contribute ancient ridge in most cases;
- From q=0.007 to 0.0001 is the third zone where SS may significantly de-spin Iapetus and eventually be captured by Saturn.

C.2. An estimate of the different time-scales of evolution of the Sub-Satellite for the given range of q=0.0001 to 1.0

In KM formulation following is the definition of evolution factor and time constant of evolution:

$$\in (evolution\ factor) = \frac{a_{present} - a_{G1}}{a_{G2} - a_{G1}} \quad C.4$$

$$\tau(time\ constant\ of\ evolution) = \frac{a_{G2} - a_{G1}}{V_{max}} \quad C.5$$

These two parameters are dependent on the mass ratio q where q is:

$$q(mass\ ratio) = \frac{M_{secondary}}{M_{primary}} \quad C.6$$

Table C.4 give these parameters of binary pairs studied by us. By inspection of Table C.4 it is seen that greater is q, shorter is the time scale of evolution from a_{G1} to a_{G2} and subsequently there is rapid evolution of the system.

Table C.4: The mass ratio(q), evolution factor(€) and time constant of evolution (τ) of some selected binary systems in ascending order of q.

Binary Pair	q	€	τ	Reference
Deimos+Mars	2.8×10^{-6}	1.66×10^{-14}	2.7×10^{20}y	Sharma et al.
Iapetus+Saturn	3.16×10^{-6}	2.17×10^{-8}	1.074×10^{16}y	Present paper
Phobos+Mars	1.17×10^{-5}	Death spiral	1.29×10^{19}y	Sharma et al.
HD49674A+b	1.07×10^{-4}	Death spiral	3.4Gy	Sharma

HD52265A+b	8.5×10^{-4}	0.394	2.42Gy	Sharma
HD147513A+b	9.6×10^{-4}	0.386	47.56My	Sharma
HD196050A+b	2.62×10^{-3}	0.866	5.65My	Sharma
HD111232A+b	8.33×10^{-3}	0.9827	1.63My	Sharma
Earth+Moon	0.012	0.687	313My	Sharma et al.
Pluto+Charon	0.125	1	37My	Sharma et al.
2M1207BD+BD	0.2	1	6 hours	Sharma
HD196885A+BD	0.2524	1	179y	Sharma
2M0535BD+BD	0.62	1	19197y	Sharma
GL86A+BD	0.784	1	20.9y	Sharma

From the studies of natural satellites such as Earth-Moon and Pluto-Charon, I make a following estimate of the scale of evolution of SS as given in the Table C.5.

Table C.5: Estimated transit time of Sub-Satellite for achieving an evolution factor of €=0.7 by scaling the time constant and transit time of Earth-Moon system.

q	Estimated Time Constant	Estimated transit time for reaching an evolution factor €=0.7
0.0001	10Gy	147Gy
0.001	1Gy	14.7Gy
0.006	800My	11Gy
0.009	500My	7.185Gy
0.012	313My	4.5Gy (this is the case of Earth-Moon)
0.021	200My	2.874Gy
0.03	100My	1.47Gy
0.04	90My	1.3Gy
0.1	31My	450My
0.125	24.8My	360My (this is the case of Charon-Pluto)

0.5	0	0
0.8	0	0

Using these transit time estimates, I derive and plot SS semi-major axis expansion with time for the whole range of q.

C.3. An estimate of the time-scale of evolution of the Sub-Satellite for the range of q=1.0 to 0.3 specifically at q=0.4.

Let us consider the case q=0.4. This is the case of hydro-dynamic instability. Hence after the impact almost abruptly, Iapetus-SS pair is formed in stable equilibrium configuration at outer Clarke's Orbit with orbital period of SS=spin period of SS=spin period of Iapetus~16 hours with zero time scale. Almost instantaneously Iapetus is de-spun from 13 hours to 16 hours and, in few hundred years because of high thermal conductivity, 16 hours hydro-static equilibrium ellipsoidal shape is frozen for the posterity. Subsequently because of de-spinning of Iapetus by Saturn, Sub-satellite's Synchronous Orbit expansion sweeps past the contemporary orbit of SS at 1.68My and SS as abruptly spirals-in and impacts into Iapetus leaving an equatorial ridge. This could be as ancient as seen today by Cassini Mission. This scenario is quite plausible and the remaining de-spinning is done by Saturn.

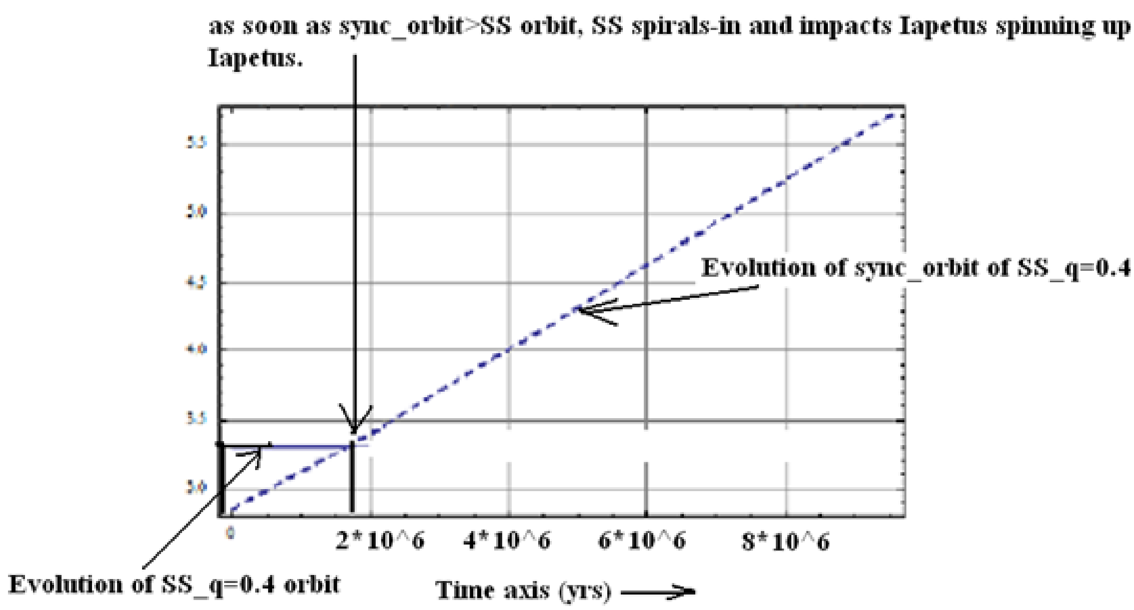

Figure C.1: Abrupt setting up of SS_q=0.4 in outer Clarke's Orbit at $3.3R_{Iap}$ and equally abrupt collapse of SS_q=0.4 orbit by spiral-in and impacting Iapetus. This spiral-in takes place due to expansion of sync-orbit of SS and becoming larger than $3.3R_{Iap}$ at 1.68My.

C.4. An estimate of the time-scale of evolution of the Sub-Satellite for the range of q=0.29 to 0.007 specifically at q=0.04

Case i: This is the classical core-accretion process by which the Sub-Satellite has been formed. At q=0.04, the constants of equation (ω/Ω) are:

$E=6.2673906 \times 10^{-10}\,m^{-1.5}$ and $F=17.768 \times 10^{-14}\,m^{-2}$.

From the (ω/Ω) equation, the two Clarke's orbits are:

Inner Clarke's Orbit=$a_{G1}=2.58R_{Iap}=1.90002 \times 10^{6}$ m which corresponds to 13 hours orbital period.

Outer Clarke's Orbit=$a_{G2}=15.55R_{Iap}=11.4368 \times 10^{6}$ m which corresponds to 191 hours=7.96 days orbital period. The MMR (2:1) is at $a_2=3.64338 \times 10^{6}$ m.

The synchronous orbit 'a_{sync}' ($2.59R_{Iap}$)>'$a_{G1}=2.58R_{Iap}$' therefore right from the beginning SS at q=0.04 is doomed to be trapped in a death spiral and make an impact at 726.4My after the debris generating impact. This SS impact will create an equatorial ridge but not an ancient one. Still this case will be analyzed as if SS is in super-synchronous orbit.

From Table C.5, the time constant of evolution=90My and the time taken to achieve (evolution factor) €=0.7 is 1.1Gy. Within these boundary conditions Structural Constant 'K' and structural exponent 'M' are determined.

At €=0.7, the semi-major axis is 8.575766×10^{6} m. Hence SS must take 1.3Gy to spiral out to a semi-major axis of 8.575766×10^{6} m. In our case SS is able to achieve evolution factor of 0.7 in 1.1Gy.

Using these boundary conditions and v_{max}=0.008m/yr, we get: 'M'=2.32072 and 'K'=2.7431×10^{27}.

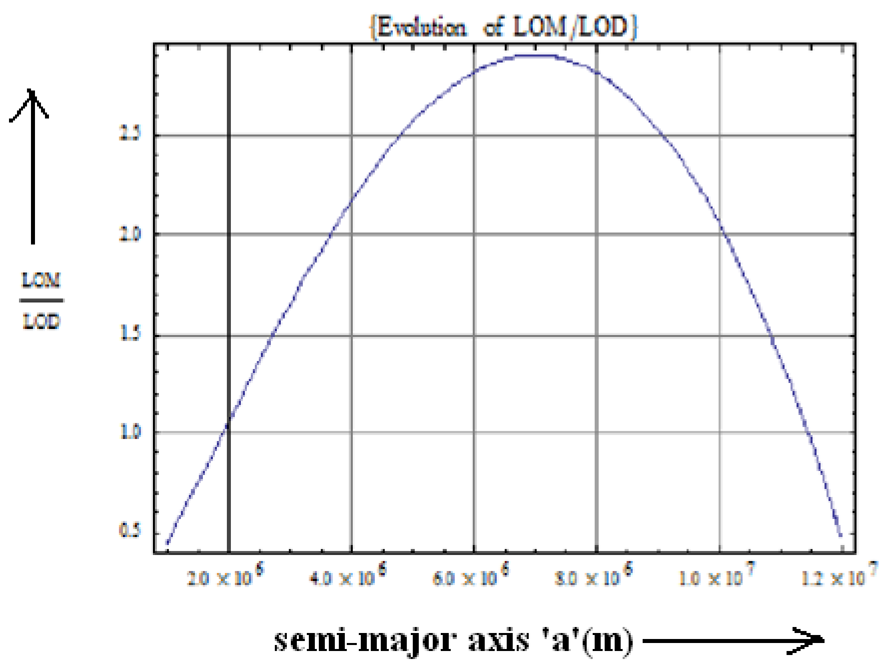

Figure C.2: Evolution of ω/Ω=LOM/LOD of SS_q=0.04 with respect to the expanding spiral orbit of SS. At $a_{G1}=1.9 \times 10^{6}$ m and at $a_{G2}=11.44 \times 10^{6}$ m, LOM/LOD=1 and orbital period=spin of SS=spin of Iapetus. At 7×10^{6} m, LOM/LOD has the maximum value of 2.9.

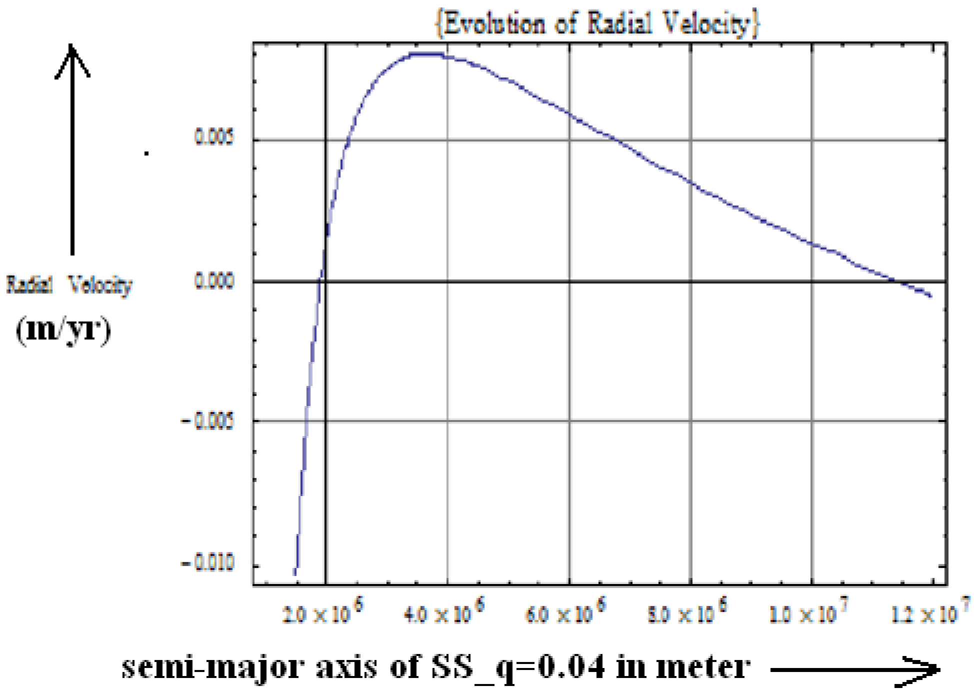

Figure C.3: The evolution of Radial Velocity as SS_q=0.04 spirals out from inner Clarke's Orbit (1.9×10^6 m) to outer Clarke's Orbit (11.44×10^6 m). The maximum radial velocity is 0.008 m/yr at $a_2=3.64338 \times 10^6$ m. This is the point where $\omega/\Omega=2$. This is also the point where the impulsive gravitational sling-shot torque ends and SS coasts on its own along an expanding spiral path.

This scenario is ruled out. This gives a much larger non-hydrostatic equilibrium anomaly. It does not help in de-spinning because SS is caught in sub-synchronous orbit and the SS will spiral-in and impact Iapetus causing it to spin up and the SS impact produces a recent equatorial ridge which is as recent as (4500M-726.4M=) 3.773Gy old.

Case ii: Consider q=0.1. Here the formation is through core accretion process hence it has an evolutionary history.

At q=0.1, the constants of equation (ω/Ω) are:

$E=9.6278003 \times 10^{-10} m^{-1.5}$ and $F=41.998 \times 10^{-14} m^{-2}$.

From the (ω/Ω) equation, the two Clarke's orbits are:

Inner Clarke's Orbit=$a_{G1}=2.56R_{Iap}=1.88649 \times 10^6$ m which corresponds to 12.44 hours orbital period.

Outer Clarke's Orbit =$a_{G2}=5.38R_{Iap}=3.96229 \times 10^6$m which corresponds to 37.87 hours=1.57 days orbital period. Here at $\omega/\Omega=2$, we get complex roots therefore we assume that v_{max} occurs at $\omega/\Omega=1.15$. The MMR (1.15:1) is at $a_2=2.35394 \times 10^6$ m.

Here also 'a_{sync}'=$2.64R_{Iap}$ > 'a_{G1}'=$2.56R_{Iap}$ therefore SS at q=0.1 is doomed to be trapped in death spiral and impact SS in 176My. This creates an equatorial ridge as ancient as 4.3Gy old still for complete physical understanding this case will be analyzed keeping SS in super-synchronous orbit.

The time constant of evolution=31My and the time taken to achieve (evolution factor) €=0.7 is 365My. Within these boundary conditions Structural Constant 'K' and Structural Exponent 'M' are determined.

At €=0.7, the semi-major axis is 3.33955×10^6 m. Hence SS must take 365My to spiral out to a semi-major axis of 3.33955×10^6 m.

Using these boundary conditions and v_{max}=0.01m/yr, 'M'=4.24294 and

'K' =4.44×10^{40}.

In Table C.6. the time evolution of SS_q=0.1 semi-major axis is given.

Table C.6: The time evolution of SS_q=0.1 semi-major axis.

Time after formation(yrs)	**Semi-major axis (a) mof SS_q=0.1**	**a (×R_{lap})**
0	1.886×10^6	2.564
130.6 M	1.9×10^6	2.58
158.05 M	1.95×10^6	2.65
169.7 M	2.0×10^6	2.71
184.7 M	2.1×10^6	2.85
196.2 M	2.2×10^6	2.99
206.5 M	2.3×10^6	3.13
216.5 M	2.4×10^6	3.26
226.7 M	2.5×10^6	3.40
237.4 M	2.6×10^6	3.53
248.9 M	2.7×10^6	3.67
261.4 M	2.8×10^6	3.81
275.3 M	2.9×10^6	3.94
290.9 M	3.0×10^6	4.08
308.8 M	3.1×10^6	4.21
329.6 M	3.2×10^6	4.35
354.1 M	3.3×10^6	4.49

383.8 M	3.4×10^6	4.62
420.7 M	3.5×10^6	4.76
468.5 M	3.6×10^6	4.89
534.4 M	3.7×10^6	5.03
636.7 M	3.8×10^6	5.17
849.6 M	3.9×10^6	5.30
889.3 M	3.91×10^6	5.32
937.6 M	3.92×10^6	5.33
999.3 M	3.93×10^6	5.34
1.08 G	3.94×10^6	5.36
1.22 G	3.95×10^6	5.37
1.61 G	3.96×10^6	5.38
3.45 G	3.96229×10^6	5.39

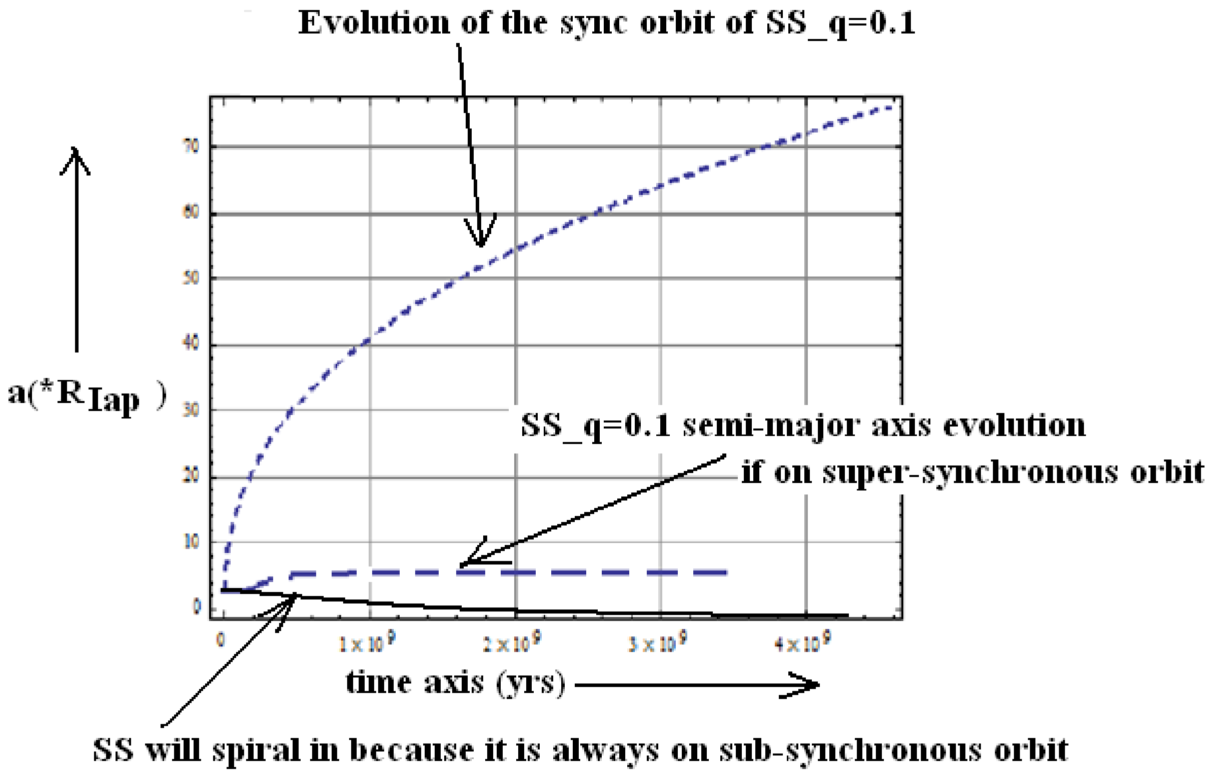

Figure C.4: The evolution of SS_q=0.1 semi-major axis in a collapsing spiral orbit.

As we see in Figure C.4, a_{syncSS}=2.64R_{Iap} for q=0.1 whereas the launching point a_{G1} of SS for q=0.1 is 2.56R_{Iap} hence SS is doomed to death spiral right from the beginning. In fact if we look at Table C.2, only for q=0.0001 to 0.006 we have a_{sync}=a_{G1}. Hence only in these cases SS can be launched in super-synchronous orbit and thereby play a role in de-spinning the Iapetus-SS system.

From q=0.007 to 0.2, in every case a_{syncSS}>a_{G1} and SS in all these cases is launched on sub-synchronous orbit where it spins-up the system.

From q=0.3 to 1, SS abruptly goes to approximately 4R_{Iap} orbit so it equally abruptly de-spins Iapetus from 13 hours to 16 hours. In few hundred years Iapetus is frozen to preserve 16 hours non-hydrostatic equilibrium anomaly to this day.

From this discussion it is clear the scenarios involving q=0.007 to 0.2 are ruled out as they donot help in de-spinning.

For the case q=0.1, if the time integral equation is solved for death spiral then we see that the impact time is 176My after SS is formed. This makes the equatorial ridge 4.325 Gy ancient. For q<0.1, the ridge will be still younger which does not concur with the observations of Cassini Mission.

Hence from KM point of view, q=0.2 to 0.007 SS is completely ruled out. Next we will examine q=0.006.

C.5. An estimate of the time-scale of evolution of the Sub-Satellite for the range of q=0.006 to 0.0001.

Case i: q=0.006

This is the classical core-accretion process by which the Sub-Satellite has been formed. At q=0.006, the constants of equation (ω/Ω) are:

$E=4.2404446 \times 10^{-10}\ m^{-1.5}$, $F=2.75534 \times 10^{-14}\ m^{-2}$, $B=347612.8\ m^{3/2}/s$.

From the (ω/Ω) equation, the two Clarke's orbits are:

Inner Clarke's Orbit=$a_{G1}=2.563R_{Iap}=1.88561 \times 10^6$ m which corresponds to 13 hours orbital period.

Outer Clarke's Orbit=$a_{G2}=321.55R_{Iap}=236.542 \times 10^6$ m which corresponds to 18266 hours=761 days orbital period. The MMR (2:1) is at $a_2=3.0455082 \times 10^6$ m.

Here 'a_{syncSS}'=$2.563R_{Iap}=a_{G1}=2.563R_{Iap}$ so here SS may be placed in super-synchronous orbit therefore this is a suitable case for studying from de-spinning point of view.

From Table C.5, the time constant of evolution=800My and the time taken to achieve (evolution factor) €=0.7 is 11Gy. Within these boundary conditions Structural Constant 'K' and Structural Exponent 'M' are determined.

At €=0.7, the semi-major axis is 166.145×10^6 m. Hence SS must take 11Gy to spiral out to a semi-major axis of 166.145×10^6 m.

Using these boundary conditions and v_{max}=2.5 m/yr, 'M'=3.37206 and 'K'=6.19396×10^{35}. Using these kinematic parameters the transit time to achieve 0.7 evolution factor is 11.2Gy.

In Table C.7. the time evolution of SS_q=0.006 semi-major axis is given.

Table C.7: The time evolution of SS_q=0.006 semi-major axis.

Time after formation(yrs)	Semi-major axis (a)m of SS_q=0.006	a ($\times R_{Iap}$)
0	$1.885419.7 \times 10^6$	2.563
602644	2.20689×10^6	3
2.62 M	2.94252×10^6	4
2.71 M	3.678150×10^6	5
3.24 M	4.413780×10^6	6
3.45 M	5.14941×10^6	7
3.66 M	5.885040×10^6	8
3.87 M	6.620670×10^6	9

5.47 M	7.3563×10^6	10
6.66 M	8.82756×10^6	12
7.85 M	10.29882×10^6	14
10.4 M	11.770080×10^6	16
13.14 M	13.241340×10^6	18
16.11 M	14.7126×10^6	20
23.4 M	17.655120×10^6	24
32.56 M	20.59764×10^6	28
43.99 M	23.540160×10^6	32
77.9 M	29.4252×10^6	40
119.33 M	35.31024×10^6	48
173.07 M	41.19528×10^6	56
252.11 M	47.08032×10^6	64
316.84 M	51.4941×10^6	70

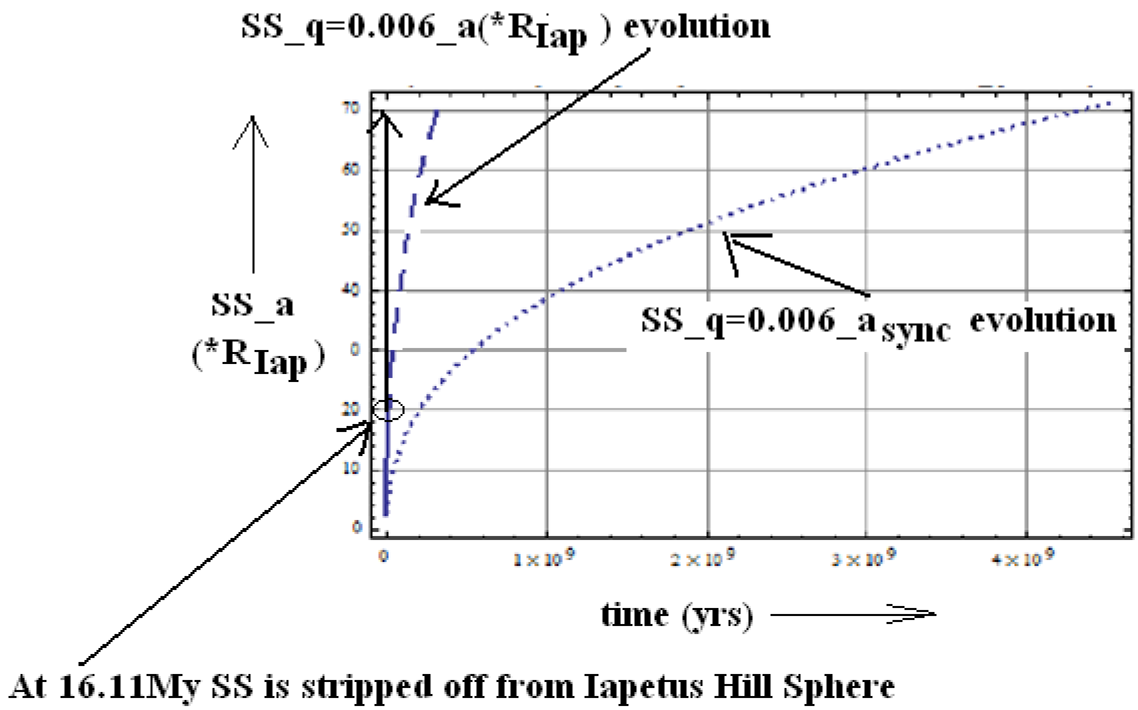

Figure C.5: Evolution of semi-major axis of SS_q=0.006. At 16.11My SS gets stripped off from Iapetus Hill Sphere and just manages to de-spin Iapetus from 13 hours to 15.77 hours.

From Figure C.5. it is evident that SS is in super-synchronous orbit. At 16.11My, SS is stripped off from Iapetus Hill Sphere. At 16.11My after the debris generating impact, SS orbital period itself de-spins from 13 hours to 283 hours because it has spiraled out from an orbit of semi-major axis $a_{G1}=1.88561 \times 10^6$ m to $a_{strip}=14.7126 \times 10^6$ m. The ratio (ω/Ω) has a value 17.9659 at $a_{strip}=14.7126 \times 10^6$ m therefore spin period of Iapetus at point of stripping is only 15.77 hours. There is insignificant de-spinning and according to this scenario the non-hydrostatic equilibrium anomaly should be much larger corresponding to 13 hours whereas it is corresponding to 16 hours. Therefore this scenario is completely rejected.

Case ii: q=0.0001

This is the classical core-accretion process by which the Sub-Satellite has been formed. At q=0.0001, the constants of equation (ω/Ω) are:

$E=3.8799531 \times 10^{-10}\ m^{-1.5}$, $F=0.046193 \times 10^{-14}\ m^{-2}$, $B=346592\ m^{3/2}/s$.

From the (ω/Ω) equation, the two Clarke's orbits are:

Inner Clarke's Orbit=$a_{G1}=2.558R_{Iap}=1.88187 \times 10^6$ m which corresponds to 13 hours orbital period.

Outer Clarke's Orbit =a_{G2} =$959050R_{Iap}=705506 \times 10^6$ m which corresponds to 2.98×10^9 hours=124.17×10^6 days orbital period. The MMR (2:1) is at $a_2=2.98813 \times 10^6$ m.

Here 'a_{sync}'=$2.558R_{Iap}=a_{G1}=2.558R_{Iap}$ so here SS may be placed in super-synchronous orbit and it is a suitable case for studying from de-spinning point of view.

From Table C.5, the time constant of evolution=10Gy and the time taken to achieve (evolution factor) €=0.7 is 147Gy. Within these boundary conditions Structural Constant 'K' and Structural Exponent 'M' are determined.

At €=0.7, the semi-major axis is 4.93855×10^{11} m. Hence SS must take 147Gy to spiral out to a semi-major axis of 4.93855×10^{11} m.

Using these boundary conditions and v_{max}=160,000,000 m/yr=1.6×10^{8} m/yr, 'M'=3.49794 and 'K'=4.09153×10^{42}. Using these kinematic parameters the transit time to achieve 0.7 evolution factor is 152Gy.

In Table C.8. the time evolution of SS_q=0.0001 semi-major axis is given.

Table C.8: The Time Evolution of SS_q=0.0001 semi-major axis.

Time after formation(yrs)	**Semi-major axis (a) m of SS_q=1×10^{-4}**	**a ($\times R_{Iap}$)**
0	1.8854197×10^{6}	2.563
0.0237	2.20689×10^{6}	3
0.0289	2.94252×10^{6}	4
0.0336	3.678150×10^{6}	5
0.0389	4.413780×10^{6}	6
0.045	5.14941×10^{6}	7
0.052	5.885040×10^{6}	8
0.061	6.620670×10^{6}	9
0.070	7.3563×10^{6}	10
0.093	8.82756×10^{6}	12
0.122	10.29882×10^{6}	14
0.157	11.770080×10^{6}	16
0.199	13.241340×10^{6}	18
0.247	14.7126×10^{6}	20

As seen in Table C.8, at orbital radius of $20R_{Iap}$=14.7126×10^{6} m, the orbital period of SS is 284.2 hours. The ratio (ω/Ω) at this radius is 17.96 therefore the spin period of Iaptus is 15.82 hours. This happens in 0.247 year. That is almost instantly Iapetus is de-spun from 13 hours to 16 hours and SS gets stripped off Iapetus Hill Sphere. From this point onward Iapetus is de-spun by Saturn.

If there was no SS then Saturn-Iapetus System would have taken 1.68My to de-spin from 13 hours to 16 hours confronting us with the problem of justifying non-hydrostatic equilibrium anomaly corresponding to 16 hours.

So this scenario is acceptable.